國立中正大學 104 學年度碩士班招生考試試題

系所別：電機工程學系- 電磁晶片組 / 計算機工程組 / 電力與電能處理甲組 / 電力與電能處理乙組

科目：線性代數與微分方程

第 2 節

1. (10%) Let $\boldsymbol{R}^4$ have the Euclidean inner product. Find two unit vectors that are orthogonal to all three of the vectors $\mathbf{u} = (1,1,-4,0)$, $\mathbf{v} = (-1,1,2,-2)$, $\mathbf{w} = (3,-2,5,4)$.

2. (10%) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, show that A is diagonalizable if $(a-d)^2 + 4bc > 0$.

3. (5%) Let $Bx = b$ be a linear system whose augmented matrix $(B|b)$ has reduced row echelon form

$$\left(\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Find all solutions to the system.

4. $d_1 = (1, 1, 1, 1)$, $d_2 = (1, 2, 3, 4)$, $d_3 = (0, 1, 0, 1)$, $d_4 = (1, 0, 1, 0)$, $d_5 = (4, 3, 2, 1)$.
   a. (5 %) Find the dimension of the space V spanned by the vectors d_1, d_2, d_3, d_4, and d_5.
   b. (5 %) Find a subset of these five vectors that forms a basis for the space V.
   c. (5 %) Express each vector di not in the basis (found in 4(b)) as a linear combination of the basis vectors.

5. Two matrices M_1 (3 by 3) and M_2 (4 by 4) have the same determinant value.

$$M_1 = \begin{bmatrix} 4 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 3 & 4 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 & m & 0 & 0 \\ m & 0 & m & 0 \\ 0 & m & 0 & m \\ 0 & 0 & m & 0 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 0 & m & 0 & 0 \\ 2m & 0 & 2m & 0 \\ 0 & 3m & 0 & 3m \\ 0 & 0 & 4m & 0 \end{bmatrix}$$

   a. (5 %) Find the value of m.
   b. (5 %) Find the determinant of the matrix M_3 (4 by 4).

Show all your work and write your answers clearly.

1. (10%) Solve the initial-value problem

$$(2e^{2x}\sin y + 2xy)dx + (e^{2x}\cos y + x^2)dy = 0$$

2. (10%) Given that $y_1 = x$ is a solution of the differential equation

$$x^2y'' - 4xy' + 4y = 0$$

find a second linearly independent solution by the *method of reduction of order* in the interval $x > 0$.

3. (10%) Find a general solution of the system

$$\mathbf{x}' = \begin{pmatrix} 9 & 4 & 0 \\ -6 & -1 & 0 \\ 6 & 4 & 3 \end{pmatrix}\mathbf{x}$$

4. (10%) Solve the initial-value problem

$$y^{(4)} + 2y'' + y = 4te^t, \quad y(0) = y'(0) = y''(0) = y'''(0) = 0$$

5. (10%) Find the eigenvalues and associated eigenfunctions of the Sturm-Liouville problem

$$\begin{aligned} &y'' + \lambda y = 0 \qquad (0 < x < L); \\ &y(0) = 0, \qquad hy(L) + y'(L) = 0 \quad (h > 0). \end{aligned}$$