Useful constants:

1. Planck's constant $h = 6.62\times10^{-34}$ J·s.
2. $hc = 1240$ eV·nm.
3. Speed of light $c = 3 \times 10^{8}$ m/s.
4. Electron charge $e = 1.6 \times 10^{-19}$ C.
5. Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Problems:

1. A ruler of 1 meter long moves parallel to its length with speed $v = 0.8c$ relative to you.
   (a) (5%) Compute the length of the ruler measured by you.
   (b) (5%) How long does it take for the ruler to pass you?

2. (10%) Assume an electron is bound to a neutron by the gravitational force to form an ion. Derive an expression for the allowed electron energies based on Bohr's model. Express your answer in terms of G, the gravitational constant, m_e and m_n, the masses of electron and neutron, respectively, and other constants.

3. (10%) At what speed would a body's relativistic energy E be twice its rest energy m_0c^2? c is the speed of the light and m_0 the rest mass of the body. Express your answer in terms of c.

4. (10%) A light source of wavelength λ illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.00 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.00 eV. What is the work function of the metal?

5. (10%) A photon of wavelength λ collides with a free electron at rest. After the collision, the electron is observed to be moving in the direction of the original photon and the photon is scattered away along an unknown direction. Find the momentum and the kinetic energy of the electron. Assume the speed of electron is close to c, the speed of light. Express your answers in terms of λ, h (the Planck constant), c, and m (mass of the electron).

6. (10%) Consider a quantum system consisting of two particles in a magnetic field $\vec{B}$. Both particles have spin of 1/2. Thus this system has four possible states, which we label schematically as (1) ↑↑, (2) ↑↓, (3) ↓↓ and (4) ↓↑. Let μ_B be the Bohr magneton, then the magnetic energy of a spin-half particle in a magnetic field $\vec{B}$ can be expressed as $E = -\vec{\mu}_B \cdot \vec{B}$. If the two particles are indistinguishable, find the probabilities of the system in each of the possible energy levels. The system is kept at temperature T.

7. (10 points) The potential energy of the two atoms in a diatomic molecule as a function of the atom-atom distance, r, can be expressed as

$$U(r) = U_0[(e^{(b-r)/a} - 1)^2 - 1]$$

where U_0, a, and b are positive constants with $b >> a$. Find the bond length and the binding energy in terms of U_0, a, and b.

8. (10%) Electrons of kinetic energy K in parallel trajectories are directed at two narrow slits a distance d apart. The direction of motion of the electrons is perpendicular to the plane consisting of both slits. Find the angle between the central maximum of the resulting interference pattern and the next maximum. Express your answer in terms of K, d, the electron mass m, and Planck constant h.

9. (10%) The energy reaching Earth from the Sun at the top of the atmosphere is 1.36×10^3 W/m^2, which is called the solar constant. Assuming that Earth radiates like a blackbody at uniform temperature, what is the equilibrium temperature of the Earth?

10. A particle is described by the wavefunction

$$\psi(x)=\begin{cases} A\cos(2\pi x/L) & \text{for } -(L/2)\le x\le(L/2) \\ 0 & \text{otherwise} \end{cases}$$

(a) (5%) Determine the normalization constant A.

(b) (5%) What is the probability that the particle will be found between $x = 0$ and $x = L/8$ if a measurement of its position is made?