1. 【Ordinary Differential Equations】 (20%)

   (a) Solve $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$　(10%)

   (b) Solve $y''' - 3y'' + 3y' - y = e^x - x - 1$　(10%)

2. 【Vector Calculus】 (20%)

   (a) Given the velocity vector $\vec{v} = \csc x\,\vec{i} + \sec x\,\vec{j}$ of a steady fluid flow. Is the flow irrotational? Is the flow incompressible?　(10%)

   (b) Evaluate the flux due to $2xz\vec{i} + yz\vec{j} + z^2\vec{k}$ passing through the part of the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the x-y plane.　(10%)

3. 【Fourier Analysis】 (20%)

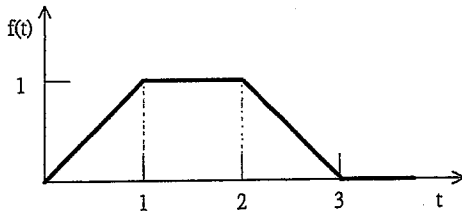   (a) Find the Fourier series of the function $f(x) = |x|$, $-\pi \le x \le \pi$　(10%)

   (b) Show that, $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \cdots = \dfrac{\pi^2}{8}$　(10%)

4. 【Laplace Transform】 (20%)

   Function f(t) is represented as the following figure:

   (a) Rewrite the function with unit step functions.　(10%)

   (b) Find its Laplace transform.　(10%)

5. 【Partial Differential Equation】 (10%)

   Solve the following PDE using the Method of Separation of Variables.
   $$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0, \quad 0 < x < a,\ 0 < y < b$$
   boundary conditions: $T(0,y) = T(a,y) = T(x,b) = 0$, $T(x,0) = f(x)$

6. 【Residue Integration】 (10%)

   Evaluate $\displaystyle\int_0^\infty \frac{x^2}{(x^2+9)(x^2+4)^2}\,dx$