國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【光電所碩士班】　　題號：435001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. Let $\mathbf{A}=\begin{bmatrix}1 & 2 & 1\\ 1 & 2 & 0\\ 2 & 1 & -1\end{bmatrix}$. Find the rank and the determinant of $\mathbf{A}^5=\mathbf{AAAAA}$. Show the details of your work. (10%)

2. Transform the quadratic form: $7x_1^2+5x_2^2+6x_3^2-4x_1x_3+4x_2x_3=210$ to principle axes (to a canonical form; i.e. the form of $\lambda_1y_1^2+\lambda_2y_2^2+\lambda_3y_3^2=210$). Express the new coordinate vector $\mathbf{y}=[y_1 \quad y_2 \quad y_3]^T$ in terms of the original coordinate vector $\mathbf{x}=[x_1 \quad x_2 \quad x_3]^T$. (15%)

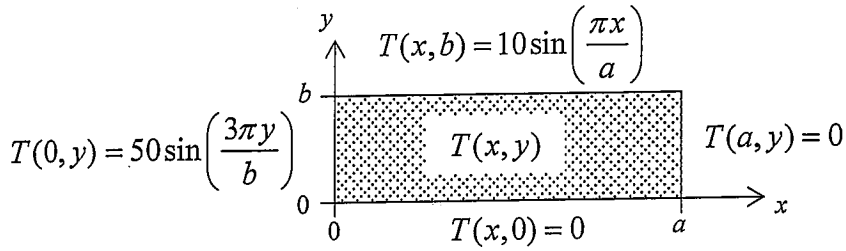
3. Find respective general solution $y(x)$ for the following ordinary differential equations. (Note: $y'=\frac{d}{dx}y(x)$, $y''=\frac{d^2}{dx^2}y(x)$, and $y'''=\frac{d^3}{dx^3}y(x)$)

   a. $y'+y-x\sqrt{y}=0$ (10%)

   b. $y'''-3y''+3y'-y=e^x+x^2$ (10%)

   c. $y''+y=\cos^2 x$ (10%)

4. Find the steady-state temperature $T(x,y)$ of the thin rectangle in the following figure, if the left side is kept at $50\sin(3\pi y/b)$, the upper side is kept at $10\sin(\pi x/a)$, and the other sides are kept at 0. (15%)



5. For a given Sturm-Liouville problem: (Note: $y'=\frac{d}{dx}y(x)$ and $y''=\frac{d^2}{dx^2}y(x)$)

$$x^2y''+xy'+\lambda y=0, \quad 1\le x\le 3$$

$$y'(1)=0 \text{ and } y(3)=0$$

   a. Find the eigenvalues λ and eigenfunctions $y(x)$. (10%)

   b. Verify orthogonality of the eigenfunctions. (Hint: rewrite the given equation in the form of $[p(x)y']'+[q(x)+\lambda\times w(x)]y=0$ to find the weight function $w(x)$) (10%)

6. Show the triangle inequality for complex numbers, $|z_1+z_2|\le|z_1|+|z_2|$ (10%)