1. (15%) Solve the general and particular solutions of the following ordinary differential equation (ODE).

$$y'\cos^2 x + 3y = 1, \qquad y(\tfrac{1}{4}\pi) = \tfrac{4}{3}$$

2. (25%) Solve the given nonhomogeneous linear ODE only by the method of variation of parameter. Show the details of your work. [Hint : use $y = u \cdot x^{-1/2}$]

$$\left(x^2 D^2 + xD + \left(x^2 - \frac{1}{4} \right) I \right) y = x^{3/2} \sin x$$

3. (15%) Find the coefficient of Fourier series in the signal $x(t) = \sin(10t)\left(\sum_{k=-3}^{3} \frac{1}{1+jk} e^{jkt} \right)$.

[Note: the coefficient of each Fourier series can be represented in the fractional form.]

4. (9%) Find the solution of the following equations only by Cramer's law.

$$8x_1 - 4x_2 + 3x_3 = 0$$
$$x_1 + 5x_2 - x_3 = -5$$
$$-2x_1 + 6x_2 + x_3 = -4$$

5. (16%) The matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$,

5.1 (6%) Please find the eigenvalues.
5.2 (6%) Please find, for each eigenvalue, the corresponding eigenvector of the matrix.
5.3 (4%) Please also find the orthogonal matrix that diagonalizes the matrix.

6. (20%) Use the Laplace transform to solve the initial value problem of the following ODE. Show the details of your work

$$y_1' = 2y_1 + y_2, \quad y_2' = 4y_1 + 2y_2 + 64tu(t-1), \quad y_1(0) = 2, \quad y_2(0) = 0,$$