國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】　　題號：434004

※本科目依簡章規定「不可以」使用計算機　　共1頁第1頁

There are 7 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.

1. Explain the following terminologies.
   (a) [3%] Partial ordering relation
   (b) [3%] Planar graph
   (c) [4%] Left coset

2. Let $A = \{a, b, c, d, e, f, g, h\}$ and R is the equivalence relation on A.
   (a) [6%] If R induces the partition $A = \{a\} \cup \{b, c\} \cup \{d, e\} \cup \{f, g, h\}$, what is R?
   (b) [6%] If R induces the partition $A = A_1 \cup A_2 \cup \ldots \cup A_n$ where n is a positive integer, then $|R| = ?$
   (c) [8%] How many relations on A are equivalence relations that satisfy $b \in [h]$?

3. [10%] Construct a state diagram for a finite state machine with the input alphabet $I = \{a, b\}$ and the output alphabet $O = \{0, 1\}$ that recognizes all strings in $\{b, a\}^*\{bb\} \cup \{a, b\}^*\{aa\}$. Note that the state diagram should contain 3 states only or get no credits.

4. Let $f(x) = x/(1-x)$ and $g(x) = 1/(1-x^2)$.
   (a) [5%] If $f(x)$ is the generating function of a sequence, what is the sequence?
   (b) [5%] If $g(x)$ is the exponential generating function of a sequence, what is the sequence?
   (c) [5%] Use $f(x)$ and $g(x)$ to express the generating function for the number of integer solutions for the equation, $c_1+2c_2 = 100$ with $c_1 \geq 0$ and $c_2 \geq 1$, such that the answer is the coefficient of x^{100}.

5. Solve the following recurrence relations:
   (a) [5%] $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$, $a_0 = -3$, $a_1 = 0$
   (b) [6%] $a_n + 2a_{n-1} - 4a_{n-2} - 8a_{n-3} = 0$, $n \geq 3$, $a_0 = 0$, $a_1 = -6$, $a_2 = 8$
   (c) [6%] $a_n - 5a_{n-1} = 2(7^n)$, $n \geq 1$, $a_0 = 10$
   (d) [8%] $a_n + 2a_{n-1} + 2a_{n-2} = 0$, $n \geq 2$, $a_0 = 1$, $a_1 = 1$. Note that the solutions should contain real numbers only or get no credits.

6. [10%] Let $G = (V, E)$ be a loop-free graph with $|V| = n \geq 2$. For all $x, y \in V$ and $x \neq y$, $\deg(x)+\deg(y) \geq n-1$. Prove that G is connected.

7. [10%] Prove that an integral domain $(T, +, \cdot)$ is a field if it is finite.