Question 1. (20%、5% for each sketch)

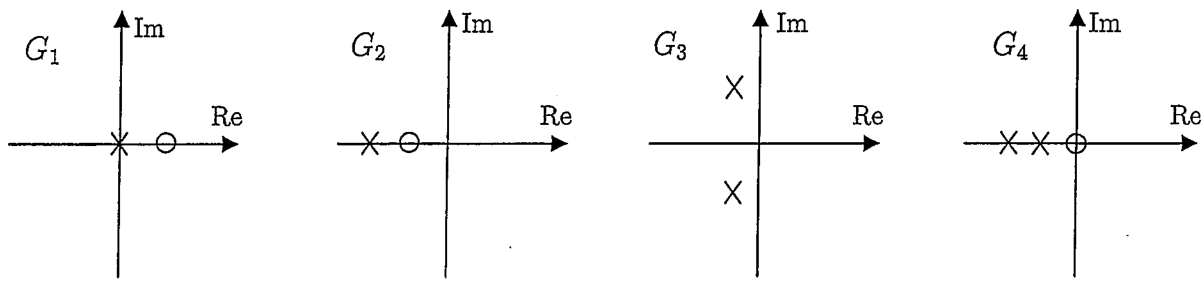

Figure 1: pole/zero plots for Question 1.

The pole/zero plots of four transfer functions $G_1(s)$ to $G_4(s)$ are shown in Figure 1. Sketch the step response of each transfer function. On your sketches, you should show important details of the responses, such as the overshoot, undershoot, steady state, etc. if any. Missing any important detail will result in mark deduction.

Question 2 (30%)

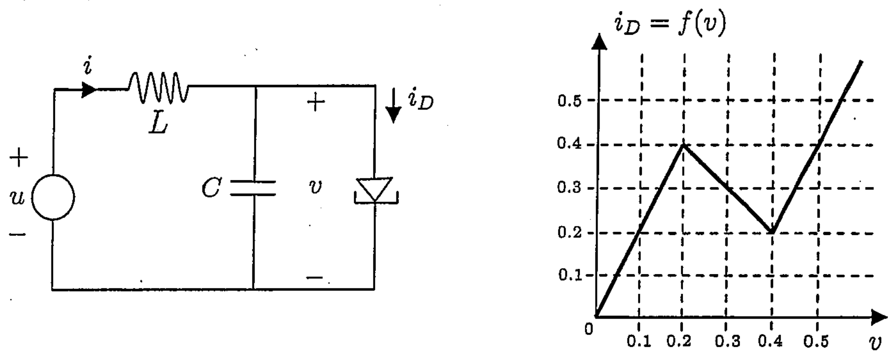

Figure 2: tunnel diode circuit for Question 2.

The left-hand side of Figure 2 shows the schematic model of an electronic circuit involving a tunnel diode. At time $t \geq 0$, let $u(t)$ denote the source voltage, $i(t)$ denote the inductor current and $v(t)$ denote the voltage across the capacitor (and thus the tunnel diode). For the sake of argument, suppose that the static non-linear current-voltage characteristic $i_D(t) = f(v(t))$ of the tunnel diode is as shown on the right in Figure 2, that the inductance $L = 1$ and that the capacitance $C = 1$. Applying Kirchoff's voltage and current laws yields the following differential equations:

$$\frac{di}{dt} = u(t) - v(t); \quad \frac{dv}{dt} = i(t) - f(v(t))$$

(a) (10%) Determine all triples (u_q, v_q, i_q) that are consistent with an equilibrium inductor current $i_q = 0.3$, where u_q and v_q denote equilibrium values for the source voltage and the capacitor voltage.

(b) (10%) For the equilibrium $(u_q, v_q, i_q) = (0.5, 0.5, 0.4)$, linearize the governing equations and derive a transfer function model to relate the Laplace transform $\Delta_v(s)$ of $\delta v(t) = v(t) - v_q$ (as an output) and the Laplace transform $\Delta_u(s)$ of $\delta u(t) = u(t) - u_q$ (as an input). Is the transfer function stable?

(c) (10%) For the transfer function identified in (b), calculate the response of the transfer function when $\delta u(t) = \sin(t)$.

Question 3 (10%)

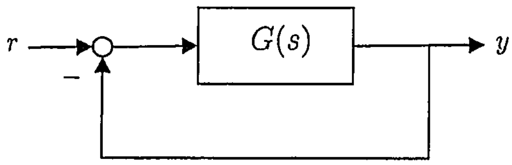

Figure 3: a feedback system for Questions 3.

Consider a standard negative feedback system shown in Figure 3, where $G(s) = \dfrac{2}{s(s+\sqrt{3})}$. Determine how many seconds loop delay $\tau > 0$ can be tolerated if a phase margin of more than 45° is required in the presence of such delay.

Question 4 (15%)

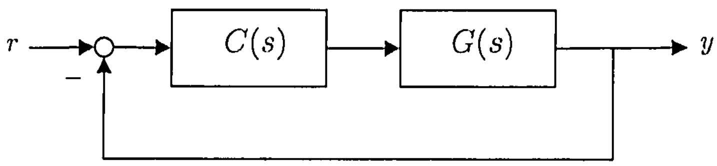

Figure 4: a feedback system for Questions 4 and 5.

Consider a standard negative feedback system shown in Figure 4, where the plant $G(s) = \dfrac{1}{s^2+5s+6}$ and the compensator $C(s) = \dfrac{K(s+10)}{s+4}$. Find the positive compensator parameter values $K > 0$, such that the following conditions both hold: (i) the closed-loop is *STABLE*; and (ii) the closed-loop impulse response has a mode that decays *SLOWER* than e^{-t}.

Question 5 (25%)

Consider the feedback system shown in Figure 4 again. Suppose now the plant has the transfer function $G(s) = \dfrac{1}{s^2-1}$.

(a) (10%) Explain why a lag compensator (e.g., a PI controller) can *NEVER* stabilize G.

(b) (15%) Find a *PROPER* controller C such that the following conditions both hold: (i) the closed-loop is *STABLE*; and (ii) the response of the closed-loop system to any step input always has *ZERO* steady state error.

End of Examination