答題時，每題都必須寫下題號與步驟。

1. (10%) Evaluate $\displaystyle\int_0^\infty \frac{\arctan x}{1+x^2}\,dx$.

2. (10%) Evaluate $\displaystyle\lim_{t\to 0^+}\left(\int_0^t \frac{dx}{\sqrt{1-x^2}}\right)^t$.

3. (10%) Evaluate $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} (1+\sqrt{x^2+y^2})\,dy\,dx$.

4. (15%) Determine the set of real numbers x for which $\displaystyle\sum_{k=1}^\infty \left(1+\frac{1}{k}\right)^k (x-1)^k$ converges.

5. (15%) Minimize $2x+y+4z$ subject to $x^2+y^2+z^2=7$.

6. (20%) Let $M_2(\mathbb{R})$ be the collection of 2×2 matrices with real-number entries and $L:M_2(\mathbb{R})\to M_2(\mathbb{R})$ be a linear transformation defined by

$$L(X)=\begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix} X - X \begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}.$$

   (a) Find the dimension of the image of L.

   (b) Find a basis for the kernel of L.

7. (20%) Let $f(x,y,z)=\lambda(x^2+y^2-z^2)+z^2+(2-4\lambda)xz$ be a quadratic form in x,y and z.

   (a) Find a 3×3 symmetric matrix A such that $f(x,y,z)=vAv^t$, where $v=[x,y,z]$ and v^t is the transpose of v.

   (b) Find the real values of λ such that f is positive definite.

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