

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (1) Find a recurrence relation for the number of bit strings of length  $n$  that do not have two consecutive 0s. Note that initial conditions should be given. (5%)  
 (2) How many such bit strings are there of length 5? (5%)  
 (3) Solve the recurrence relation. (5%)

2. (1) Find the smallest relation containing the relation  $\{(1,2), (1,4), (3,3), (4,1)\}$  on  $U = \{1, 2, 3, 4\}$ , that is  
 (a) reflexive, symmetric, and transitive. (5%)  
 (b) Reflexive, antisymmetric and transitive. (5%)

- (2) Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?  
 (a)  $K_5$ , (b)  $K_6$ , (c)  $K_{3,3}$ , (d)  $K_{3,4}$  (Hint:  $K_n$  denotes complete graph;  $K_{a,b}$  denotes complete bi-partite graph.) (5%)

3. (1) Given a computer network, as shown below, where the nodes are computer centers, and the weight of each edge is the response time (in seconds) between the two end nodes. Find a route with the shortest response time between the node A and node Z.

	A	B	C	D	E	F	Z
A	0	3	5	-	7	-	-
B	3	0	-	4	5	-	-
C	5	-	0	4	5	6	-
D	-	4	4	0	6	-	-
E	7	5	5	6	0	4	3
F	-	-	6	-	4	0	2
Z	-	-	-	-	3	2	0

(10%)

- (2) Is there a Hamilton cycle in the computer network? Is there an Euler cycle or Euler path? (10%)
4. (1) Let  $G$  be the grammar with alphabet  $\{0, 1\}$ , starting symbol  $S$ , and productions  $P = \{S \rightarrow 11S, S \rightarrow 0\}$ . Describe  $L(G)$ , the language of this grammar. (5%)  
 (2) Given the state transition table as follows. ( $S_0$  is the start state.  $S_0$  and  $S_4$  are "accept" states.)  
 (a) Find the nondeterministic finite-state automaton. (10%)  
 (b) Find a deterministic finite-state automaton that recognizes the same state transition table. (10%)

state	Next state	
	Input = 0	Input = 1
$S_0$	$S_0, S_2$	$S_1$
$S_1$	$S_3$	$S_4$
$S_2$	Not defined	$S_4$
$S_3$	$S_3$	Not defined
$S_4$	$S_3$	$S_3$

5. (1) Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$  and the ordering of elements of  $U$  has the elements in increasing order. That is  $a_i = i$ . Assume that we represent  $U$  as a bit string. We represent a subset  $A$  of  $U$  with the bit string of length 8, where the  $i$ th bit in the string is 1 if  $a_i$  belongs to  $A$  and is 0 if  $a_i$  does not belong to  $A$ . The bit strings for the sets  $M = \{1, 2, 3, 4, 5\}$  and  $N = \{1, 3, 5, 7\}$  are 11111000, and 10101010, respectively. Find the union and intersection of  $M$  and  $N$  and represent them using bit strings. (5%)
- (2) Consistent system specifications should not contain conflicting requirements which would be used to derive a contradiction. Determine if the following specifications are consistent:  
 "The message is stored in the router's buffer or it is sent."  
 "The message is not stored in the router's buffer."  
 "If the message is stored in the router's buffer, then it is sent."  
 (Hint: you can express the specifications using logic expressions. Let  $p$  denote "the message is stored in the router's buffer". Let  $q$  denote "the message is sent." Try to find an assignment of  $p$  and  $q$  to make the three specifications to be all true.) (5%)
- (3) A program segment is said to be partially correct with respect to the initial assertion  $p$  and the final assertion  $q$  if whenever  $p$  is true for the input values of  $S$  and  $S$  terminates, then  $q$  is true for the output values of  $S$ . This can be denoted as  $p\{S\}q$ . Consider the program segment " $a := 3; n := a + m;$ " given the initial assertion  $p: m := 1$ . Find the final assertion  $q$ . (5%)
6. (1) The algorithm for evaluating a polynomial  $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  at  $x = c$  is as following:  
**Procedure polynomial**( $c, a_0, a_1, \dots, a_n$ : real numbers)  
 power := 1  
 y :=  $a_0$   
**for**  $i := 1$  **to**  $n$   
**begin**  
     power := power \*  $c$   
     y :=  $y + a_i$  \* power  
**end.**  
 How many multiplications and additions are used to evaluate a polynomial of degree  $n$  at  $x = c$ ? (Hint: Do not count additions used to increment the loop variable.) (5%)
- (2) A committee of three members decides issues for a company. Each member votes either yes or no for each proposal. A proposal is passed if there are at least two yes votes. Design a logic circuit that determines if a proposal is passed. (5%)