國立成功大學 104 學年度碩士班招生考試試題

系所組別：工程科學系甲組

考試科目：線性代數與機率

考試日期：0212，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A system is described by $\mathbf{z}=\mathbf{Hx}+\mathbf{v}$, where $\mathbf{z}\in R^{m\times 1}$ is the observation vector, the system matrix $\mathbf{H}\in R^{m\times n}$, $m\ge n$ and $\text{rank}(\mathbf{H})=n$, $\mathbf{x}\in R^{n\times 1}$ is the fixed system state, and $\mathbf{v}\in R^{m\times 1}$ is the uncorrelated Gaussian noise vector with variance σ_v^2, i.e. $\mathbf{v}\sim N(\mathbf{0}_{m\times 1},\boldsymbol{\Sigma}_v)$ has zero mean $\mathbf{0}_{m\times 1}$, which is an $m\times 1$ vector with all-zero elements, and covariance matrix $\boldsymbol{\Sigma}_v=\sigma_v^2\mathbf{I}_{m\times m}$, where $\mathbf{I}_{m\times m}$ is the $m\times m$ identity matrix. Based on the observation vector, the system state is assumed to be estimated by $\hat{\mathbf{x}}=(\mathbf{H}^T\mathbf{WH})^{-1}\mathbf{H}^T\mathbf{Wz}$, where $\mathbf{W}=\boldsymbol{\Sigma}_v^{-1}$ is the inverse matrix of $\boldsymbol{\Sigma}_v$.
   (a) Let's define the residue vector $\mathbf{r}=\mathbf{z}-\mathbf{H}\hat{\mathbf{x}}$. Find the mean vector and covariance matrix of $\mathbf{r}$. The covariance matrix should be expressed by $\boldsymbol{\Sigma}_v$, $\mathbf{W}$, and $\mathbf{H}$. Hint: the covariance matrix of a zero-mean random vector $\mathbf{b}$ is $E[\mathbf{bb}^T]$. (10 pt.)
   (b) If the system is attacked by inserting an attack vector $\mathbf{a}=\mathbf{Hc}$, where $\mathbf{c}\in R^{n\times 1}$ is an arbitrary non-zero vector, into the observation vector as $\mathbf{z}_a=\mathbf{z}+\mathbf{a}=\mathbf{H}(\mathbf{x}+\mathbf{c})+\mathbf{v}$. Under $\mathbf{z}_a$, find the estimated system state $\hat{\mathbf{x}}_a=(\mathbf{H}^T\mathbf{WH})^{-1}\mathbf{H}^T\mathbf{Wz}_a$. The estimated system state should be expressed by $\boldsymbol{\Sigma}_v$, $\mathbf{W}$, $\mathbf{H}$, $\mathbf{x}$, $\mathbf{c}$, and $\mathbf{v}$. (5 pt.)
   (c) Find the residue under attack by $\mathbf{r}_a=\mathbf{z}_a-\mathbf{H}\hat{\mathbf{x}}_a$. The residue should be expressed by $\boldsymbol{\Sigma}_v$, $\mathbf{W}$, $\mathbf{H}$, and $\mathbf{v}$. (5 pt.)
2. Let P^2 be the set of polynomials over R with powers less than 3. Prove $\{1,\ (x-3),\ (x-3)^2\}$ is a base of P^2. (20 pt.)
3. A communication system transmits binary information over a channel that introduces random bit error for both binary 0 and 1 with probability *p*=0.01. The transmitter transmits each information bit three times:

   0→000;　　　　1→111,

   and a decoder takes a majority vote of the received bits to decide on what the transmitted bit was. Find the probability that the receiver will make an incorrect decision. (20 pt.)
4. A current with the Rayleigh probability density function

$$f_I(i)=\begin{cases}(i/a^2)\exp(-i^2/2a^2), & i>0\\ 0, & \text{otherwise}\end{cases}$$

   Is passed through a resistor with a resistance of $2\pi\ \Omega$. If the mean value of the current is $E[I]=a(\pi/2)^{1/2}=2$ Amps and the mean square current $E[I^2]=2a^2$, what is the mean value of the power dissipated in the resistor? Express your answer using a constant. (20 pt.)
5. A transistor may come from any one of three manufactures A, B, and C with probabilities $P_A=0.25$, $P_B=0.5$, and $P_C=0.25$, respectively. The probabilities that the transistor will be defective in manufactures A, B, and C are 0.01, 0.02, and 0.03, respectively.
   (a) Find the probability that a randomly selected transistor will be defective. (10 pt.)
   (b) If the chosen transistor is defective, what is the probability that this transistor comes from manufacture B? (10 pt.)