

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the problem $x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = \ln x$ (25%)

2. If all eigenvalues λ of the following matrix A are $\lambda \geq 0$.

Questions:

(a) What is the range of k . (9%)

(b) Following the part (a) to find the maximum eigenvalue and the minimum eigenvalue of A . (8%)

(c) Find the eigenvector corresponding to the maximum eigenvalues for positive k . (8%)

$$A = \begin{bmatrix} 4 & k & 0 \\ k & 3 & k \\ 0 & k & 2 \end{bmatrix}$$

3. Solve the problem $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$, $x \geq 0$, $0 \leq y \leq 1$ with the boundary conditions

$$T(x, 0) = T(x, 1) = 0 \text{ and } \lim_{x \rightarrow \infty} T(x, y) = 0. (25\%)$$

4. Calculate $\oint_C \frac{dz}{(z^2 + 1)(z - 2i)^2}$ in which C is the circle $|z + i| = 4$. (25%)