

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (40 %) A second order mechanical system is described as following:
 $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = f(t)$ and $\dot{y}(0) = y(0) = 0$. Answer the following questions:
 - (a) Find the transfer function of above system. (5 points)
 - (b) If $f(t) = \delta(t) = \text{Derac delta function}$, please find the response $y(t)$. (5 points)
 - (c) If $f(t) = e^{-t} + e^t$, please find the response $y(t)$. (10 points)
 - (d) If $f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$, Please find the response $y(t)$. (10 points)
 - (e) Plot (roughly) the response $y(t)$ of (d). (10 points)

2. (10%) Find the convolution integral, $\int_0^t e^{-\tau} \sin(t - \tau) d\tau$. (10 points)

3. (10%) Please explain its physical meaning of Divergence theorem of Gauss.

$$\iiint_T \text{div} F dV = \iint_S F \cdot n dA \quad (10 \text{ points})$$

4. (10%) Let $v = [yz \quad 3zx \quad z] = yzi + 3zxj + zk$
 - (a) $\text{div } v = ?$ (divergence of a vector field). (5 points)
 - (b) $\text{curl } v = ?$ (curl of a vector field). (5 points)

5. (10%) Find the directional derivative of $f(x, y, z) = x^2 + y^2 + z^2$ at P: (2, 2, -1) in the direction of $a = [1 \quad 1 \quad 3]$. (10 points)

6. (20%) For a matrix A, please answer the following,
 - (a) Write down the definitions of eigenvector and eigenvalue (5 points)
 - (b) Write down the procedure of how to obtain the eigenvalues and eigenvectors. (10 points)
 - (c) What is the definition of nonsingular matrix? (5 points)