※ 考生請注意：本試題不可使用計算機

1. Consider the following linear system:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

(a) Please derive the "Separation Principle" for it. (10%)
(b) What's the physical meaning of "Separation Principle"? (10%)

2. For the plant

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{(s+1)(s+2)}$$

Please design the feedback gain to yield a 15% overshoot and the settling time at 0.3 second. (15%)

3. Consider the closed-loop system represented in the state space:

$$\dot{X} = AX + Br$$
$$y = CX$$

(a) If the input is a unit step, where $r = 1$, please derive the steady state error e_{ss}. (10%)
(b) If the input is a ramp function, where $r = t$, please derive the steady state error e_{ss}. (10%)

4. Consider the state equation of a second-order digital control system that is represented by

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k)$$

where

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

Please find the control gain matrix G for the state feedback controller $u(k) = -Gx(k)$ such that the characteristic roots of the closed-loop system are $z_1 = -0.1$ and $z_2 = 0.5$. (10%)

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5. Consider the differential equation

$$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+7=f(x)$$

where $f(x)=\sin x$ is the input. Please linearize the differential equation for small excursions, (a) $x=0$, and (b) $x=\pi$. (15%)

6. Please find out the transfer function of the Bode plot with a damping which damping ratio is 0.2 as below (20%)