國立成功大學 104 學年度碩士班招生考試試題

系所組別：航空太空工程學系甲乙丙丁組、民航研究所甲組

考試科目：工程數學　　考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Prove that rank(**A**)=rank($\mathbf{A}^T$) for every $m \times n$ matrix. (20%)
2. Derive the integration formula (20%)

$$\int_0^\infty \frac{\cos(ax) - \cos(bx)}{x^2} dx = \frac{\pi}{2}(b-a) \qquad (a \geq 0, b \geq 0)$$

Then, with the aid of the trigonometric identity $1 - \cos(2x) = 2sin^2x$, point out how it follows that

$$\int_0^\infty \frac{sin^2 x}{x^2} dx = \frac{\pi}{2}$$

3. Use the method of separation of variables to solve the problem: (20%)

$$u_t = 9u_{xx} \qquad 0 < x < 8, t > 0$$

$$u(0,t) = u(8,t) = 0, \qquad t > 0$$

$$u(x,0) = x + 2, \qquad 0 < x < 2$$

4. Solve the following initial value problem: (20%)
$y'' + 4y' + 8y = e^{2t} - 2\delta(t - 2\pi), \quad y(0) = 2, \quad y'(0) = 0$

5. Let $\vec{R}(s)$ be a space curve represented by the arc length parameter s. (20%)

(a) What is the unit tangent vector $\vec{T}(s)$ along the space curve? (5%)

(b) Represent the unit normal vector $\vec{N}(s)$ along the space curve by use of $\vec{T}(s)$ and the curvature κ of the curve. (5%)

(c) Let $f(x,y,z) = x^2 + y^2 - z$ be a temperature field. What is the rate of change of $f(x,y,z)$ at the point $P(1,1,2)$ on the space curve $\vec{R}(s)$ in the direction of the vector $\vec{v} = 2\vec{i} + 2\vec{j} + \vec{k}$. (5%)

(d) In problem (c), what is the maximum value of the rate of change of $f(x,y,z)$ at the point $P(1,1,2)$? (5%)