國立成功大學 104 學年度碩士班招生考試試題

系所組別：測量及空間資訊學系

考試科目：工程數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. The Fourier series of a function $f(x)$ with period 2π is given by

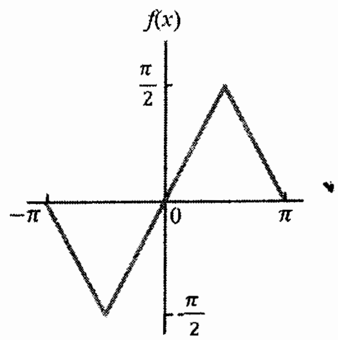
$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right),$$

where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx,,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx, \quad n = 1, 2, \cdots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx, \quad n = 1, 2, \cdots$$



(a) Find the Fourier series of the function $f(x)$ (10 %).

(b) Explain the meaning of the obtained coefficients a_n and b_n. (5 %)

2. Give a nonhomogeneous equation and its corresponding homogeneous equation

$$y'' + p(x)y' + q(x)y = r(x) \qquad (1)$$

$$y'' + p(x)y' + q(x)y = 0 \qquad (2)$$

Prove that

(a) The difference of two solutions of Eq. (1) is a solution of Eq. (2). (7 %)

(b) The sum of a solution of Eq. (1) and a solution of Eq. (2) is a solution of Eq. (1). (8 %)

3. (a) Solve the least-squares linear system **Ax=b**. (15%).

(b) Explain the geometric meaning of least-squares solutions. (5%)

$$\mathbf{A} = \begin{bmatrix} 1 & 5 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix}$$

4. (a) Find the singular value decomposition (SVD) of matrix **A**. (15%)

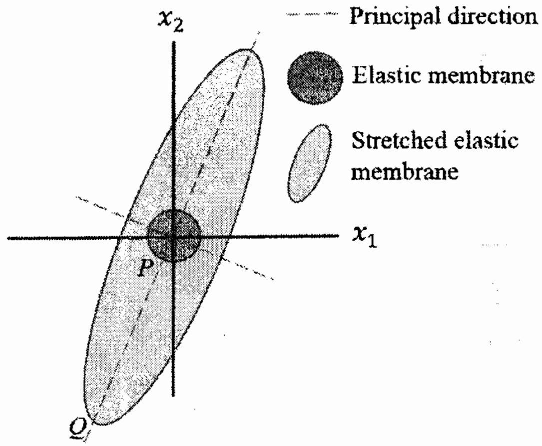
(b) Introduce an application of SVD. (5%)

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

5. An elastic membrane (具有彈性的薄膜) in the x_1x_2-plane with boundary circle $x_1^2 + x_2^2 = 1$ (see the figure) is stretched so that a point $P: (x_1, x_2)$ goes over into the point $Q: (y_1, y_2)$ given by

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{A}\mathbf{x} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Find the principal directions of the deformed membrane. (15%)



6. Given four vectors $v_1 : [1 \quad 3 \quad 2 \quad 5]^T$, $v_2 : [1 \quad 5 \quad 4 \quad 0]^T$, $v_3 : [0 \quad 1 \quad 1 \quad 0]^T$, $v_4 : [-1 \quad 5 \quad 2 \quad 8]^T \in R^4$, then, (answer 'Yes' or 'No') (15%)

(a) The vector set is linearly independent.

(b) The vector set cannot form a basis for R^4.

(c) The matrix $\mathbf{A} = [v_1 \quad v_2 \quad v_3 \quad v_4]$ is invertible.

(d) The rank of the matrix $\mathbf{A}$ is 3.

(e) $\mathbf{Ax}$=0 has only the trivial solution, that is, $\mathbf{x}$=0

(f) The transformation matrix $\mathbf{A}$ is a one-to-one transformation.

(g) The transformation matrix $\mathbf{A}$ is a onto transformation