國立成功大學 104 學年度碩士班招生考試試題

系所組別：機械工程學系甲乙丙丁戊組

考試科目：工程數學　　　　考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. **(15%) If** $A=\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$**, and an orthogonal matrix** P **can diagonalize** A **to a diagonal matrix** D **such that** $D=P^{T}AP$**, find** P **and** D **as well as calculate** A^{100}.

2. **Solve the following ODEs:**

(10%) (2.1) $y''+4y'+4y=2e^{-2x}$

(10%) (2.2) $x^2y''+xy'-y=x^2e^x$

3. (20%) Solve, by use of **Laplace transform**, the one-dimensional diffusion problem formulated below:

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial x^2} \qquad (0<x<\infty,\ t>0);$$
$$u=U\cdot H(t) \qquad (x=0),$$
$$u\to 0 \qquad (x\to\infty),$$
$$u=0 \qquad (t=0),$$

where $u=u(x,t)$, ν and U are constant, and $H(t)$ is the Heaviside unit-step function.

Hint: For $f(t)=\mathrm{erfc}\left(\frac{a}{2\sqrt{t}}\right)$, with $\mathrm{erfc}(\eta)=1-\frac{2}{\sqrt{\pi}}\int_0^{\eta}e^{-\xi^2}d\xi$ being the complementary error function, the Laplace transform $F(s)=\int_0^{\infty}e^{-st}f(t)\,dt=\frac{\exp(-a\sqrt{s})}{s}$.

4. (15%) The inverse Laplace transform can be written as

$$f(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st}F(s)\,ds,$$

where the path of integral with respect to s is a vertical line parallel to the imaginary axis, and is on the right of all the singularities of $F(s)$, in the complex s plane.

Now, by use of the **residue theorem**, calculate the inverse Laplace transform of $\frac{s^3}{s^4-a^4}$.

5. **(15%)** Determine the solution T(x, t) of the following boundary value problem.

$$\frac{\partial^2 T}{\partial t^2}=\frac{\partial^2 T}{\partial x^2}+T, \quad 0<x<\pi \quad \text{and} \quad t>0$$

with the boundary conditions

$$T(0,t)=T(\pi,t)=0$$

and the initial conditions

$$T(x,0)=\begin{cases} x, & 0<x<\pi/2 \\ \pi-x, & \pi/2\le x<\pi \end{cases}, \qquad \frac{\partial T}{\partial t}(x,0)=0$$

6. **(15%)** Determine the solution T(r, θ) of the following boundary value problem.

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0, \quad 0<\theta<\pi \quad \text{and} \quad 0<r<1$$

with the boundary conditions

$$T(r,0)=T(r,\pi)=0, \qquad 0<r<1$$

and

$$T(1,\theta)=1, \qquad 0<\theta<\pi$$