

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please solve the following equations: (5 points for each one)

A. $y''+2y'+y = e^{-x}$

B. $y'''+2y''-y'-2y = \sin 3x$

C. $x^2y''+xy'-y = \ln x$

D. $y(t) + 2\int_0^t y(\eta)\cos(t-\eta)d\eta = 4e^{-t} + \sin t$

2. Please solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with the following boundary conditions: (15 points for each one)

A.
$$\begin{cases} u(0, y) = 0, & u(5, y) = 1 - y, & 0 < y < 2 \\ \frac{\partial u}{\partial y}\Big|_{y=0} = 0, & \frac{\partial u}{\partial y}\Big|_{y=2} = 0, & 0 < x < 5 \end{cases}$$

B.
$$\begin{cases} \frac{\partial u}{\partial x}\Big|_{x=0} = 0, & 0 < y < \pi \\ u(x, 0) = 0, & \frac{\partial u}{\partial y}\Big|_{y=\pi} = 0, & x > 0 \end{cases}$$

3. Please solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + \sin 2\pi x$ with $\begin{cases} u(x, 0) = \sin \pi x, & 0 < x < 1 \\ t > 0, & u(0, t) = 0, & u(\pi, t) = 0 \end{cases}$ (15 points)

4. Trapezoidal method is used to evaluate the integral $\int_a^b f(x)dx$, that is, $\int_a^b f(x)dx = \frac{(b-a)}{2}[f(a) + f(b)] + E$,

where E is the error term. Please derive the error term E in terms of $(b-a), f(x)$ and its derivatives evaluated at midpoint $\bar{x} = \frac{(a+b)}{2}$. (15 points)

5. Two finite difference methods $\frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2} + P(x_i)\frac{y_{i+1} - y_{i-1}}{2\Delta x} + Q(x_i)y_i = R(x_i)$ and

$\frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2} + P(x_i)\frac{y_i - y_{i-1}}{\Delta x} + Q(x_i)y_i = R(x_i)$ are used for the second-order linear differential equation

$\frac{d^2 y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$, please derive the truncation errors for each method. (20 points)