編號： 46 國立成功大學 104 學年度碩士班招生考試試題

系所組別：光電科學與工程學系甲、乙組

考試科目：工程數學 考試日期：0212，節次：3

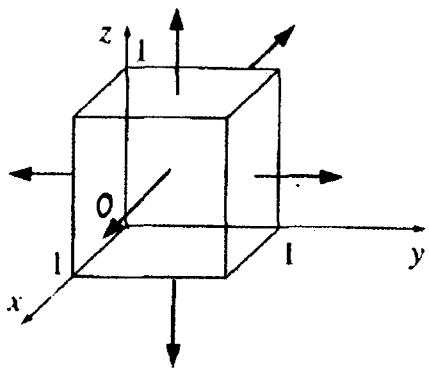
※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the directional derivative of $f(x,y,z)=2x^2+3y^2+z^3$ in the direction of $\vec{x}-2\vec{y}$ at point (1,1,1). (10%)

2. Solve the following system of differential equations by diagonalization. (15%)

$$\frac{dx}{dt} = 3x + 3y$$
$$\frac{dy}{dt} = 1x + 5y$$

3. What is the outward flux of the vector field $\vec{F} = y^3\vec{x} + (3xy + z^3)\vec{y} + (3yz)\vec{z}$ through the surface of the square box shown below? (10%)



4. Solve $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$ subject to the boundary conditions $\psi(0) = 0,\ \psi(L) = 0$. ($\hbar$ and m are constants) (10%)

5. Find the inverse of $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. (5%)

6. (1) Suppose $y(t)$ is a function for which $y'(t)$ is piecewise continuous and of exponential order c. Please justify $y(0) = \lim_{s\to\infty} sY(s)$, where $Y(s)$ is the Laplace transform of $y(t)$. (2%)

(2) Consider the initial-value problem $ty'' + y' + ty = 0,\ y(0) = 1,\ y'(0) = 0$. Please find $Y(s)$. (12%)

7. Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$ in Fourier series, and write the converging value at x=0 for the Fourier series. (12%)

8. Use Residue theorem to evaluate $\int_0^\infty \frac{x^{p-1}}{1+x}dx, 0 < p < 1$. (12%)

9. (1) Find the eigenvalues and eigenfunctions of the boundary-value problem

$$x^2y'' + xy' + \lambda y = 0,\ y(1) = 0,\ y(3) = 0. \quad (8\%)$$

(2) Put the differential equation in self-adjoint form. (2%)

(3) Give an orthogonality relation. (2%)