編號： 47

國立成功大學 104 學年度碩士班招生考試試題

系所組別：光電科學與工程學系甲、乙組

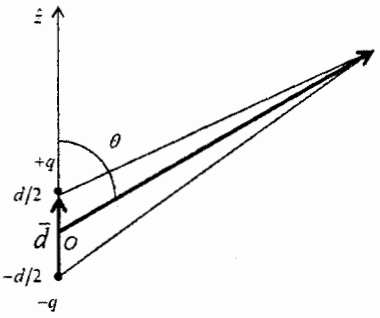
考試科目：電磁學

考試日期：0212，節次：2

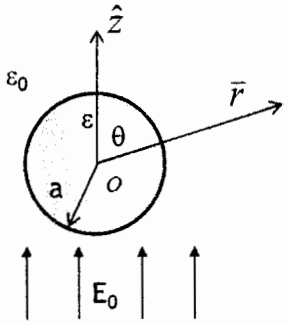
※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) For a pair of equal and opposite charges q and –q separated by a small distance d, find the electric field **E** at location $\vec{R}$ $(|\vec{R}| \gg d)$ in terms of $\vec{p} = q\vec{d}$ and $\vec{R}$ (using

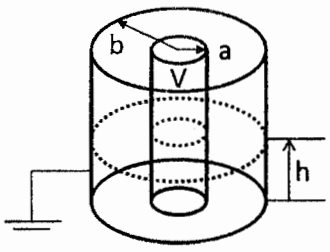
$$\left|\vec{R} \pm \vec{d}/2\right|^{-3} = \left|\left(\vec{R} \pm \vec{d}/2\right)\cdot\left(\vec{R} \pm \vec{d}/2\right)\right|^{-3/2} = \left|R^2 \pm \vec{R}\cdot\vec{d} + \left(\tfrac{d}{2}\right)^2\right|^{-3/2} \simeq \left|R^2 \pm \vec{R}\cdot\vec{d}\right|^{-3/2} \simeq R^{-3}\left(1 \mp \tfrac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^2}\right)$$



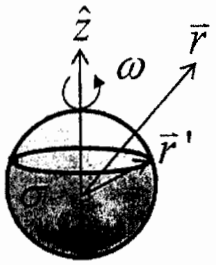
Problem 1



Problem 2



problem 3



problem 4

2. (15%) For a dielectric sphere with radius **a** placed in a uniform electric field $\vec{E} = E_0\hat{z}$. Using the continuity boundary conditions, find
   (a) the potential Φ inside (r<a) and outside (r>a) the sphere
   (b) electric field **E** inside (r<a) and outside (r>a) the sphere
   (c) Find the polarizability α of the sphere where α is defined as $P = \varepsilon_0\alpha E_0$

   (Expand Φ in $\Phi(r,\theta) = \sum_{l=0}^{\infty}\left[A_l r^l + B_l r^{-(l+1)}\right]P_l(\cos\theta)$ and $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$)

3. (15%) For a coaxial cable with inner radius a and outer radius b
   (a) (5%) find the capacitance C per unit length
   (b) (5%) find the self-inductance L per unit length
   (c) (5%) the coaxial cable is vertically stand upright and is maintained at potential V at inner cylinder r=a and grounded at outer radius r=b. Find the height **h** that the oil (with dielectric susceptibility χ_e and mass density ρ) will rise inside the region between a<r<b.

4. (10%)
   (a) (5%) Derive ampere's law $\nabla \times B = \mu J$ in terms of vector potential **A**
   (b) (5%) For a spherical shell of radius **R** with a uniform surface charge σ spinning along $\hat{z}$ axis at angular velocity ω which carries a surface current density $\vec{J}_s = \sigma\vec{v} = \sigma\vec{\omega}\times\vec{r}$, find the total dipole moment $\vec{m}$ of the spherical shell.

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5. (20 %) the E-field of a uniform plane wave propagating in a dielectric medium is given by $\mathrm{E}(t,z) = \boldsymbol{a}_x cos\left(\frac{10^9}{2\pi}t - \frac{z}{\sqrt{3}}\right) - \boldsymbol{a}_y sin\left(\frac{10^9}{2\pi}t - \frac{z}{\sqrt{3}}\right)$

(5-a) (5 %) Determine the frequency and wavelength of the wave.

(5-b) (5 %) What is the dielectric constant of the medium?

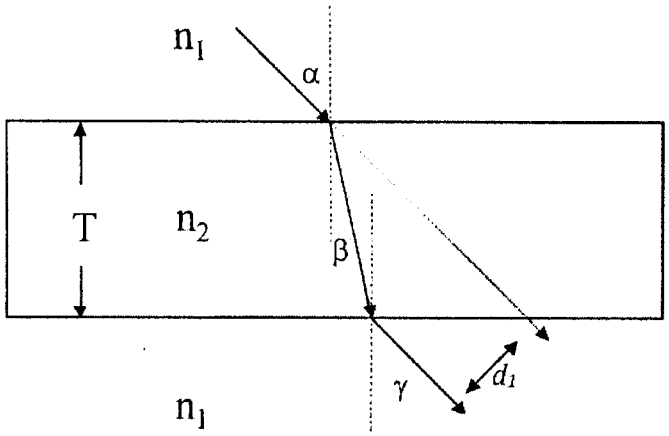
(5-c) (5 %) Describe the polarization of the wave.

(5-d) (5 %) Find the corresponding H-field.

6. (10 %) (Oblique incidence of plane waves at plane boundaries)

(6-a) (5 %) From Fresnel's equations (Reflection coefficient for parallel polarization) $\Gamma_{\parallel} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 cos\theta_t - \eta_1 cos\theta_i}{\eta_2 cos\theta_t + \eta_1 cos\theta_i}$, and $\mu_1 = \mu_2$, please derive Brewster angle ($\theta_{B\parallel} = tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$) of no reflection for the case of parallel polarization.

(6-b) (5 %) A light ray is incident from air (n_1) obliquely on a transparent sheet thickness T with an index of refraction, n_2, as shown below. The angle of incidence is α. Find the lateral displacement (d_1) of the emerging ray.



7. (10 %) Determine the dominant and their frequencies in an-air-filed rectangular cavity resonator for (a) a>b>d, (b) a>d>b, and (c) a=b=d, where a, b, and d are the dimensions in the x-, y-, and z-directions, respectively.

8. (10 %) A thin quarter-wavelength vertical antenna over a perfectly conducting ground is excited by a time-harmonic source at its base. Find its radiation pattern, radiation resistance(R_r), and directivity(D). Here, the quarter-wave antenna radiates only into the upper half-space, its total radiated power is $P_r = 18.27 I_m^2$.