國立成功大學 104 學年度碩士班招生考試試題

系所組別：物理學系
考試科目：近代物理學
考試日期：0212，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A neutron with kinetic energy K enters a nucleus, and experiences an external potential energy V = 0 at the nuclear surface, and a very rapidly dropping internal potential energy $V = -V_0$, as illustrated in the Figure 1. Considering the scattering process as a one dimensional step potential, please calculate the reflection coefficient R in terms of K and V_0. (15%)

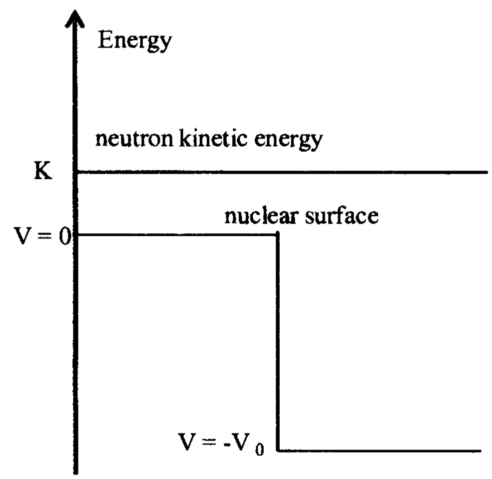

Figure 1

2. (a) Please derive the Maxwell's speed distribution law:

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}$$

from Boltzmann distribution $$f(E) = Ae^{-E/kT}$$

where M is the molar mass of the gas, R is the gas constant, T is the gas temperature, v is the molecular speed, and E is the gas energy. (10%)

(b) use the result to find the average speed v_{avg} (5%),

(c) use the result to find the root-mean-square speed v_{rms} (5%)

(d) use the result to find the most possible speed v_p (5%)

3. In an elastic scattering event, as shown in Figure 2, the scattering vector **q** is defined as $\mathbf{q} = \mathbf{k}_f - \mathbf{k}_i$, where $\mathbf{k}_i$ and $\mathbf{k}_f$, with the vector length k, are the incident and scattering wavevectors respectively. From the Born approximation:

$$\psi_1 = -\frac{e^{ikr}}{r}\frac{2\mu}{\hbar^2(4\pi)}\int V(\mathbf{r}')e^{i\mathbf{q}\cdot\mathbf{r}'}d^3\mathbf{r}' = \frac{e^{ikr}}{r}f(\theta,\phi)$$

where μ is the reduced mass.

Show that (a), (if we only focus on spherically symmetric, i.e., $V(\mathbf{r}) = V(\mathrm{r})$) (10%)

$$f(\theta) = -\frac{2\mu}{\hbar^2}\int \frac{\sin qr'}{q} V(r') r' dr'$$

here $q = 2k \sin(\theta/2)$, the vector length of $\mathbf{q}$

(b) calculate $f(\theta)$, for the potential: (10%)

$$V(r) = -\frac{zZe^2 e^{\left(-\frac{r}{a}\right)}}{r}$$

where ze and Ze represent the charges of incident particle and target respectively.

(c) use the results from (b) to calculate the scattering differential cross section (5%)

$$\frac{d\sigma}{d\Omega} \equiv |f(\theta)|^2$$

(d) show that when $a \to \infty$, we can recover the classical Rutherford scattering differential cross section (5%)

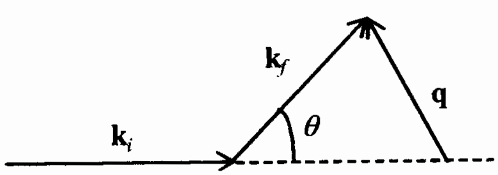

Figure 2

4. Evaluate

(a) $\exp\left(\frac{ia\hat{p}_x}{\hbar}\right) f(x)$, where a is a constant (10%)

(b) if ψ is real, show that $\langle \hat{p}_x \rangle = \int \psi^* \hat{p}_x \psi dx = 0$ (10%)

5. calculate the commutators:

(a) $[\hat{x}, \hat{L}^2]$ (5%)

(b) $[\hat{p}_x, \hat{L}^2]$ (5%)