國立成功大學104學年度碩士班招生考試試題

系所組別：物理學系

考試科目：物理數學

考試日期：0212，節次：1

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. We have a matrix $M = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 2 & 3 & 1 \\ 3 & 0 & 0 & 2 \\ 1 & 3 & 2 & 1 \end{vmatrix}$. (a) Calculate the determinant of M. (b) If $\{\lambda_i, i = 1, 2, 3, 4\}$ are the eigenvalues of M, calculate $\sum_{i=1}^{4} \lambda_i$ and $\sum_{i=1}^{4} \lambda_i^2$. (4+3+3)

2. Calculate $\oint_C (x^2 + yz^2)dx + (2x - y^3)dy$, where C is a circle $(x-2)^2 + (y-3)^2 = 9$ on the $z = 0$ plane. (10)

3. (a) Show by the Wronskian method that the functions $\{x^n/n!, n = 0, 1, \cdots, \infty\}$ are linearly independent.
   (b) Construct from $\{x^n\}$ the first three Laguerre polynomials $L_0 = 1$, $L_1 = 1 - x$, $L_2 = (2 - 4x + x^2)/2$ in $0 \le x < \infty$ by the Gram-Schmidt orthogonalization procedure with the weight function $w(x) = exp(-x)$ and normalization $\langle L_m | L_n \rangle = \delta_{mn}$. (5+10)

4. A particle of unit mass ($m = 1$) is initially at $x(0) = A$ with velocity $\dot{x}(0) = 0$.
   (a) $d^2x/dt^2 + \gamma\, dx/dt + kx = 0$, find $x(t)$ for all time $t > 0$ for $k > \gamma^2$, $k = \gamma^2$ & $k < \gamma^2$ respectively.
   (b) $d^2x/dt^2 + \gamma\, dx/dt + kx = F\cos(\omega t)$, where F is an external driving force. Assume you have obtained the steady state solution $B\cos(\omega t + \phi)$, i.e. $B = B(F, \gamma, k)$ & $\phi = \phi(\gamma, k)$ are already known, find $x(t)$ for all time $t > 0$ for $k > \gamma^2$. (9+6)

5. (a) Find the poles & residues of $\dfrac{1}{1+z^n}$.
   (b) Derive $\int_0^\infty \cos(t^2)dt = \int_0^\infty \sin(t^2)dt = \dfrac{\sqrt{\pi}}{2\sqrt{2}}$。 (Hint: Take the contour in the fig. 5. Note that $f(z) = \exp(iz^2)$ has no singularity in the finite complex plane.) (10+10)

6. Bessel functions obey the recurrence relation $J_{n+1}(x) = -J_n'(x) + \dfrac{n}{x}J_n(x)$. Show by mathematical induction（數學歸納法 " if correct for n, then also correct for $n+1$ "）that $J_n(x) = (-1)^n x^n \left(\dfrac{1}{x}\dfrac{d}{dx}\right)^n J_0(x)$ for any integer n. (Hint: Calculate $J_n'(x)$, the derivative of $J_n(x)$.) (15)

7. Show that the inverse Fourier transform of $G(\vec{k}) = \dfrac{1}{(2\pi)^{3/2}k^2}$ is $g(\vec{r}) \equiv \dfrac{1}{(2\pi)^{3/2}}\int G(k)e^{i\vec{k}\cdot\vec{r}}d^3\vec{k} = \dfrac{1}{4\pi r}$, where $k \equiv |\vec{k}|$ & $r \equiv |\vec{r}|$. (Hint: $\int_0^\infty \dfrac{\sin x}{x}dx = \dfrac{\pi}{2}$.) (15)

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