編號： 39　　　**國立成功大學 104 學年度碩士班招生考試試題**

系所組別：數學系應用數學碩士班

考試科目：線性代數　　　　考試日期：0212，節次：2

第 1 頁，共 1 頁

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Note: $\mathbb{R}$ denotes the field of real numbers.

1. (10%) Are the vectors $(1,1,1,0)$, $(0,0,1,1)$, and $(2,2,-1,-3)$ linearly independent in $\mathbb{R}^4$? *Justify your answer.*

2. (15%) Find all possible real numbers x_1, x_2, x_3, and x_4 that satisfy the following system of linear equations:

$$\begin{array}{rcrcrcrcr} 3x_1 & + & 4x_2 & - & 2x_3 & + & 7x_4 & = & -2 \\ x_1 & + & 3x_2 & + & x_3 & + & 4x_4 & = & 1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 4x_4 & = & -2 \end{array}$$

3. (15%) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator defined by

$$T(x_1,x_2,x_3) = (2x_1 - 3x_2 - 4x_3, -x_1 + 4x_2 + 4x_3, x_1 + x_2 + x_3).$$

Does there exist an ordered basis β for $\mathbb{R}^3$ such that the matrix representation $[T]_\beta$ of T with respect to β is a diagonal matrix? *Justify your answer.*

4. (15%) Let A be a real 5×5 matrix satisfying $A^3 - 4A^2 + 5A - 2I = O$, where I is the 5×5 identity matrix and O is the 5×5 zero matrix. Is the matrix $A^4 + A^3 - 3A^2 - 3A$ invertible? *Justify your answer.*

5. (15%) A square matrix A is called an orthogonal matrix if A^tA is the identity matrix, where A^t is the transpose of A. Prove that every real 2×2 orthogonal matrix is either

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

for some real number θ.

6. (15%) Determine all inner products $\langle\cdot,\cdot\rangle$ on $\mathbb{R}^2$ such that

$$\langle (a,b), (-b,a) \rangle = 0$$

for all real numbers a and b.

7. (15%) Let T be a linear operator on a finite-dimensional real inner product space V. Prove that if T is self-adjoint (i.e., T is its own adjoint), then there exists an orthonormal basis for V consisting of eigenvectors of T.