編號： 213

國立成功大學 104 學年度碩士班招生考試試題

系所組別：電機資訊學院-資訊聯招

考試科目：計算機數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Prove that if A is nonsingular then A^T is nonsingular and $(A^T)^{-1} = (A^{-1})^T$. (10%)
2. Analyze the complexity of evaluating the determinant of an $n \times n$ matrix by cofactors. (10%)
3. Determine which of the following sets forms a subspace of R2.(multiple answers) (5%)
   (a) $\{(x_1, x_2)^T | x_1 x_2 = 0\}$
   (b) $\{(x_1, x_2)^T | x_1 + x_2 = 0\}$
   (c) $\{(x_1, x_2)^T | x_1 = 2x_2\}$
   (d) $\{(x_1, x_2)^T | {x_1}^2 = {x_2}^2\}$
   (e) $\{(x_1, x_2)^T | |x_1| = |x_2|\}$
4. Consider the vectors $\cos(x + \alpha)$ and $\sin x$ in $C[-\pi, \pi]$. For what values of α will the two vectors be linearly dependent? (5%)
5. Let A and B be 6×5 matrices. If dim *N*(A)=2, what is the rank of A? If the rank of B is 4, what is the dimension of *N*(B)? (5%)
6. Let $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and let L be the linear operator that rotates vectors in R^2 by 45° in the counterclockwise direction. Find the matrix representation of L with respect to the ordered basis $[\mathbf{u}_1, \mathbf{u}_2]$. (5%)
7. Decompose the matrix $A = \begin{bmatrix} 2 & 5 & 4 \\ 6 & 3 & 0 \\ 6 & 3 & 0 \\ 2 & 5 & 4 \end{bmatrix}$ by singular value decomposition. (10%)
8. Let $\Sigma = \{0, 1\}$ and $A = \{0, 01, 11\} \subseteq \Sigma^*$. For $n \geq 1$, let a_n count the number of strings in A^* of length n. Find and solve a recurrence relation for a_n. (10%)
9. Let A=$\{a, b, c, d, e\}$,
   (a) How many closed binary operations f on A satisfy $f(a, b) \neq c$?
   (b) How many closed binary operations f on A have an identity and $f(a, b)=c$?
   (c) How many f in (b) are commutative?
   (d) Determine the number of relations on A that are reflexive and symmetric but not transitive.
   (e) Determine the number of equivalence relations where $b \in [e]$.
   (Note: Values of Stirling number of the second kind: S(4, 2)=7, S(4, 3)=6, S(5, 2)=15, S(5, 3)=25) (20%)
10. (a) Find the number of permutations of 0, 1, 2, 3, …, 8 in which none of the patterns '1234', '76', '23', '81' occurs. (b) How many three-element subsets of S = {1, 2,…, 10} contains no consecutive integers? (20%)