第／頁，共／頁
※ 考生請注意：本試題不可使用計算機。 請於答案卷（卡）作答，於本試題紙上作答者，不子計分。

1．$(17 \%)$ Solve

$$
\frac{\partial^{2} z}{\partial t^{2}}=4\left(\frac{\dot{\partial}^{2} z}{\partial x^{2}}+\frac{\partial^{2} z}{\partial y^{2}}\right)
$$

for $0<x<2 \pi, 0<y<2 \pi, t>0$ with following initial and boundary conditions．
$z(x, 0, t)=z(x, 2 \pi, t)=0$ for $0<x<2 \pi, t>0$ ，
$z(0, y, t)=z(2 \pi, y, t)=0 \quad$ for $0<y<2 \pi, t>0$, $\mathrm{z}(\mathrm{x}, \mathrm{y}, 0)=0$ for $0<\mathrm{x}<2 \pi, 0<\mathrm{y}<2 \pi$ ， $\frac{\partial z}{\partial t}(x, y, 0)=1$ for $0<x<2 \pi, 0<y<2 \pi$ ．

2．（15\％）Consider Bessel function $J_{v}(x)$ of order $v \geq 0$ with $j_{n}$ as its $n$－th zero．Prove that the functions $\sqrt{x} J_{v}\left(j_{n} x\right)$ ，for $n=1,2,3, \ldots$ ，are orthogonal on $[0,1]$ in the sense that

$$
\int_{0}^{1} x J_{\nu}\left(j_{n} x\right) I_{v}\left(j_{m} x\right) d x=0 \quad \text { if } n \neq m
$$

3．（15\％）Find a general solution of the following differential equation $y^{\prime \prime}-2 y^{\prime}+y=e^{x} \sin x$

4．$(18 \%)$ Find the following initial value problem

$$
\begin{gathered}
y^{I V}+3 y^{\prime \prime}-4 y=0 \\
y(0)=0, y^{\prime}(0)=-20, y^{\prime \prime}(0)=0, y^{\prime \prime \prime}(0)=80
\end{gathered}
$$

5．Evaluate the Cauchy principal value of
（a） $\int_{-\infty}^{\infty} \frac{1}{x^{4}+1} d x(10 \%)$
（b） $\int_{0}^{\infty} \frac{x \sin x}{x^{2}+9} d x(10 \%)$
6．Use contour integration to evaluate
$\int_{0}^{\infty} \frac{(\ln x)^{2}}{x^{2}+1} d x \quad(15 \%)$

