※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1.(10%) A group of 10 people sits in a round table. How many different seating arrangements are there? We assume that a seating arrangement is determined by who sits next to whom and not by where they sit. We also agree not to distinguish between clockwise and counterclockwise; all that matters is who your two neighbors are, not who is on your left and who is on your right.

2. (15%) What are the following recursive definitions describing for:

(a) the empty string λ is in set X. If string x is in set X, so are 1x0 and 0x1. If string x and string y are in X, so is xy.

(b) $f(\lambda)= \lambda$. If s has one or more symbols, write s=ra where a is a symbol and r is a string (possibly empty). Then f(s)=f(ra)=af(r).

(c) λ is a palindrome. Any symbol a is a palindrome. If x and y are palindromes, then yxy is a palindrome.

3. (10%) Suppose each object in a set X of n objects is assigned a color from a set C of r colors. Then there is a subset U with at least m objects of the same color. Please show the number m.

4. (10%) Give a big-Θ estimate of $\log_2 n^{7n+1}$.

5. (10%) If A is late then B is late. If both A and B are late, then the class is boring. Suppose that the class is not boring. What can you conclude about A?

6. (15%) The IPv4 address is a 32-bit number which divided into two parts: the network ID specifies the local network and the host ID specifies the particular computer. In order to accommodate the various sizes of the local networks connected through the Internet, the network IDs are further divided into several classes. In every class, a host ID may not consist of all 0's and or all 1's. In class A, the left-most 8 bits give the full network ID where the left-most bit is set to 0. However, neither 00000000 nor 01111111 is allowed as a network ID for a class A IP address. In class C, the left-most 24 bits give the full network ID where the left-most three bits are set to 110. The remaining bits are used for individual host IDs.

(a) Write the following IP address in dotted decimal form. 11001010 00111000 01101011 11101110

(b) How many Class A networks can there be?

(c) How many host IDs can there be for a Class A network.

7. (10%) What does the following pseudo-code generate.

     Input: A connected network N (n>2)

     Output: T

     T<-- e1, where e1 is the shortest edge of N

     while T does not contain all of N's vertices

       e <-- the shortest edge between a vertex in T and a vertex not in T

       Add edge e and the new vertex to T

     end

8. (20%) (a) What is the output d of the following algorithm?

     Input: integers m, n

     Output: d

     $d \leftarrow m; e \leftarrow n$

     while $e \neq 0$ do

       $r \leftarrow d \bmod e$

       $d \leftarrow e$

       $e \leftarrow r$

     end

(b) Is the time complexity $\Omega(n)$? Explain it.

(c) Is the time complexity $O(n)$? Explain it.

(d) Is the time complexity $O(\log_2 n)$? Explain it.