

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A unity feedback control system has the open-loop transfer function $G(s) = \frac{A}{s(s + \zeta\omega_n)}$. (20%)

(a) Compute the sensitivity of the closed-loop transfer function to changes in the parameter A . (5%)

(b) Compute the sensitivity of the closed-loop transfer function to changes in the parameter ζ . (5%)

(c) Compute the sensitivity of the closed-loop transfer function to changes in the parameter ω_n . (5%)

(d) If the unity gain in the feedback path changes to a value of positive $\beta \neq 1$, compute the sensitivity of the closed-loop transfer function to changes in the parameter β . (5%)

2. Suppose a function \tilde{A} is defined by $\tilde{A}(\mathbf{M}_{n \times m}) \equiv \mathbf{A}_{n \times n} \mathbf{M}_{n \times m} + \mathbf{M}_{n \times m} \mathbf{B}_{m \times m}$ and η is an eigenvalue of \tilde{A} . Please show that $\eta_k = \eta_{ij} = \lambda_i + \mu_j$, for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$, where λ_i and μ_j are the eigenvalues of \mathbf{A} and \mathbf{B} , respectively. (10%)

3. (a) For a linear time-varying system, please give the definition that an impulse response matrix $\mathbf{G}(t, \tau)$ is realizable. (4%)

(b) Please prove the theorem that a $q \times p$ impulse response matrix $\mathbf{G}(t, \tau)$ is realizable if and only if $\mathbf{G}(t, \tau)$ can be decomposed as $\mathbf{G}(t, \tau) = \mathbf{M}(t)\mathbf{N}(\tau) + \mathbf{D}(t)\delta(t - \tau)$, for all $t \geq \tau$, where \mathbf{M} , \mathbf{N} , and \mathbf{D} are, respectively, $q \times n$, $n \times p$, and $q \times p$ matrices for some integer n . (16%)

4. Realize the lag-lead compensator $G_c(s) = \left(\frac{s+0.1}{s+0.01}\right)\left(\frac{s+1}{s+10}\right)$ with a passive network and an active network, respectively. (25%)

5. Using Laplace transform methods, solve for the state-transition matrix, the state vector, and the output $y(t)$ of the following system for a step input $u(t)$: (25%)

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \quad y = [4 \quad 3] \mathbf{x}; \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$