

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

計算題：每題 10 分，合計 100 分

1. Evaluate $\lim_{x \rightarrow 3} \frac{x}{x-3} \cdot \int_3^x \frac{\sin t}{t} dt$

2. Find the values of the constants a and b such that $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-1}{x} = \frac{5}{3}$.

3. For what values of the constants a and b is $(1,3)$ a point of inflection of the curve $y = ax^3 + bx^2$.

4. Evaluate $\int \frac{dx}{\sqrt{1-e^{-2x}}}$

5. Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dx dy$.

6. Evaluate $\int_1^\infty \frac{\tan^{-1} x}{x^2} dx$ or show that it is divergent.

7. Find the absolute maximum and minimum values of the function

$$f(x, y) = 4xy^2 - x^2y^2 - xy^3$$

on the set D the closed triangular region in xy -plane with vertices $(0,0)$, $(0,4)$ and $(4,0)$.

8. Find a power series representation for the function $f(x) = x^2 \cdot \tan^{-1}(x^3)$ and determine the interval of convergence.

9. Let R be the region in the first quadrant bounded by the curve $y = x^3$ and $y = 2x - x^2$. Find the volume obtained by rotating R about the y -axis.

10. Find the area of the region that lies inside the cardioid $r = 1 + \cos \theta$ and outside the circle $r = 3 \cos \theta$.