編號： 257

國立成功大學 104 學年度碩士班招生考試試題

系所組別：統計學系

考試科目：數學

考試日期：0212，節次：1

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A function f is defined for all x as follows:

$$f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational;} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

Let $Q(h) = f(h)/h$ if $h \neq 0$.

(a) Prove that $Q(h) \to 0$ as $h \to 0$. (5 %)

(b) Prove that f has a derivative at 0, and compute $f'(0)$. (5 %)

2. Evaluate the integrals.

(a) $\int \sqrt{2x+1}dx$ (5 %)

(b) $\int (2+3\boldsymbol{x}) \sin 5\boldsymbol{x}\boldsymbol{dx}$ (5 %)

(c) $\int \frac{(x^2+1-2x)^{1/5}}{1-x} d\boldsymbol{x}$ (5 %)

3. Determine the derivative $f'(x)$. In each case, the function f is assumed to be defined for all real x for which the given formula for $f(x)$ is meaningful.

(a) $f(x) = \sin[\sin(\sin x)]$ (5 %)

(b) $f(x) = \sqrt{x+1} - \log(1+\sqrt{x+1})$ (5 %)

(c) $f(x) = (\log x)^x$ (5 %)

4. Evaluate the limits.

(a) $\lim_{x \to a^+} \frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$ (5 %)

(b) $\lim_{x \to 1} x^{1/(1-x)}$. (5 %)

5. The matrix C is defined as

$$C = \begin{pmatrix} 1/3 & -2/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ -2/3 & -2/3 & 1/3 \end{pmatrix}.$$

Find all the eigenvalues of C and its inverse matrix. (10 %)

6. Give the following two quadratic forms,

$$\mathbf{x}^\top A\mathbf{x} = (x_1 + 2x_2)^2 + (x_1 + 3x_2)^2 + (x_1 + x_3)^2;$$

$$\mathbf{x}^\top B\mathbf{x} = (6x_1 - 4x_3)^2 + (x_1 - 2x_2)^2 + (3x_2 - x_3)^2.$$

   (a) Identify 3 by 3 symmetric matrices A and B (10 %)

   (b) Show that A is positive definite and B is positive semi-definite. (10 %)

7. For the matrix B below, find a scalar t such that $B + tI$ is positive definite. (10 %)

$$B = \begin{pmatrix} -3 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & -2 \end{pmatrix}.$$

8. Let A be an $n \times n$ symmetric matrix such that $A^t = A^{t+1}$ for some positive integer t, $t > 2$. Show that A is an idempotent matrix, i.e. $A^2 = A$. (10 %)