

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Describe the nullspaces of these three matrices  $A$ ,  $B$ , and  $C$ :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix} \quad B = \begin{bmatrix} A \\ 2A \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 8 \\ 2 & 4 \\ 6 & 16 \end{bmatrix} \quad C = [A \ 2A] = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 3 & 8 & 6 & 16 \end{bmatrix}$$

2. (20%) Find the largest possible number of independent vectors among:

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

3. (20%) Write  $A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$  as  $S\Lambda S^{-1}$  where  $\Lambda$  is the eigenvalue matrix. Multiply  $Se^{At}S^{-1}$  to find the matrix exponential  $e^{At}$  and check  $e^{At}$  when  $t = 0$ .

4. (10%) Let  $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$  and  $A^\infty = \begin{bmatrix} 0.6 & 0.6 \\ 0.4 & 0.4 \end{bmatrix}$ , explain why the powers of  $A^k$  approach  $A^\infty$ .

(15%) Which Markov matrices produce the steady state  $(0.6 \ 0.4)$ ?

5. (20%) How can you quickly compute the four components of  $F\mathbf{c}$  starting from  $c_0 + c_2$ ,  $c_0 - c_2$ ,  $c_1 + c_3$  and  $c_1 - c_3$ ?

$$F\mathbf{c} = \begin{bmatrix} c_0 + c_1 + c_2 + c_3 \\ c_0 + ic_1 + i^2c_2 + i^3c_3 \\ c_0 + i^2c_1 + i^4c_2 + i^6c_3 \\ c_0 + i^3c_1 + i^6c_2 + i^9c_3 \end{bmatrix}$$