國立交通大學104學年度碩士班考試入學試題

科目：機率論(4082) 考試日期：104年2月6日 第2節

系所班別：統計學研究所 組別：統計所 第 頁,共 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. If $EX = 75$, $EY = 75$, $VarX = 10$, $VarY = 12$, $Cov(X,Y) = -3$, give an upper bound to
   (a) (10 %) $P(|X - Y| > 15)$
   (b) (10%) $P(X > Y + 15)$

2. The number of eggs laid on a tree leaf by an insect of a certain type is a Poisson random variable with parameter λ. However, such a random variable can be observed only if it is positive, since if it is 0 then we cannot know that such an insect was on the leaf.
   (a) (10 %) What is the expected number of eggs found on a leaf?
   (b) (10 %) What is the probability that more than ten eggs are found?

3. (20 %) Let U be uniform on (0, 1). Find the distribution of the random variable $X = [nU] + 1$ where [x] stands for the largest integer that is less than or equal to x and n is a fixed positive integer.

4. A model proposed for NBA basketball supposes that when two teams with roughly the same record play each other, then the number of points scored in a quarter by the home team minus the number scored by the visiting team is approximately a normal random variable with mean 1.5 and variance 6. In addition, the model proposes that the point differentials for the four quarters are independent.
   (a) (10 %) What is the probability that the home team wins?
   (b) (10 %) What is the conditional probability that the home team wins given that it is behind by 5 points at half time?

5. The amount of time that a certain type of component functions before failing is a random variable with probability density function $f(x) = 2x \quad 0 < x < 1$. Once the component fails it is immediately replaced by another one of the same type. If we let X_i denote the lifetime of the ith component to be put in use, then $S_n = \sum_{i=1}^{n} X_i$ represents the time of the nth failure. The long term rate r at which failures occur is defined by $r = \lim_{n\to\infty} \frac{n}{S_n}$.
   (a) (10 %) Determine r.
   (b) (10 %) How many components would one need to have on hand to be approximately 90% certain that the stock will last at least 35 days? (Hint: $P(Z < 1.284) \approx 0.9$)