1 – 10 題為選擇題 請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

1. (5%) Which matrix is invertible?

(A) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$　(B) $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$　(C) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$　(D) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

(E) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

2. (5%) Find the matrix which is NOT diagonalizable.

(A) $\begin{bmatrix} 6 & 6 \\ -2 & -1 \end{bmatrix}$　(B) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$　(C) $\begin{bmatrix} -1 & 2 & -1 \\ 0 & -3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$　(D) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(E) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & -1 \\ 2 & -1 & -1 \end{bmatrix}$

3. (5%) For any $n \times n$ matrices A and B, select the false statement below.
(A) A is invertible if and only if $\det A \neq 0$.
(B) $\det AB = (\det A)(\det B)$
(C) $\det(A+B) = \det A + \det B$
(D) $\det A^T = \det A$
(E) If A is invertible, then $\det A^{-1} = \frac{1}{\det A}$

4. (5%) Which of the statements below are correct?

(A) For any matrix A. the matrices A^TA and AA^T are positive definite.
(B) For any invertible matrix A, the matrices A^TA and AA^T are positive definite.
(C) If A is positive definite, then there exists a positive definite matrix B such that $B^2 = A$.
(D) The sum of A and B in the last statement is a positive definite matrix.
(E) None of the above.

5. (5%) Let Q be an $n \times n$ orthogonal matrix. Which of the statements below are correct?

(A) Q^{-1} is also an orthogonal matrix.
(B) $QQ^T = I_n$ (I_n is an $n \times n$ identity matrix.)
(C) Q is invertible.
(D) If λ is an eigenvalue of Q, then λ is 1 or -1.
(E) None of the above.

6. (5%) Which of the following subset of $\mathbb{R}^3$ is linearly independent?

(A) $\{[\,1\ 0\ 0\,]^T, [\,0\ 1\ 0\,]^T, [\,1\ 0\ 1\,]^T\}$
(B) $\{[\,1\ 3\ 5\,]^T, [\,2\ 4\ 6\,]^T\}$
(C) $\{[\,1\ 0\ 0\,]^T, [\,0\ 1\ 0\,]^T, [\,0\ 0\ 1\,]^T [\,1\ 0\ 1\,]^T\}$
(D) $\{[\,1\ 2\ 3\,]^T, [\,2\ 4\ 6\,]^T\}$
(E) None of the above.

7. (5%) Which of the following subset of $\mathbb{R}^3$ is an orthogonal basis?

(A) $\{[1,0,0]^T, [0,1,-1]^T, [0,1,1]^T\}$
(B) $\{[1,0,0]^T, [0,0,1]^T, [0,0,0]^T\}$
(C) $\{[3,2,0]^T, [-2,3,0]^T, [0,0,3]^T\}$
(D) $\{[1,1,1]^T, [1,-1,0]^T, [1,1,-2]^T\}$
(E) None of the above.

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8. (5%) Which of the following is a linear transformation?

(A) $T:\mathbb{R}^2\to\mathbb{R}^3, T(\begin{bmatrix}x\\y\end{bmatrix})=\begin{bmatrix}0\\x^2\\y\end{bmatrix}$　(B) $T:\mathbb{R}^3\to\mathbb{R}^3, T(\begin{bmatrix}x\\y\\z\end{bmatrix})=\begin{bmatrix}x+1\\x-y\\z\end{bmatrix}$

(C) $T:\mathbb{R}^2\to\mathbb{R}^3, T(\begin{bmatrix}x\\y\end{bmatrix})=\begin{bmatrix}x-y\\x\\y\end{bmatrix}$　(D) $T:\mathbb{R}^3\to\mathbb{R}^3, T(\begin{bmatrix}x\\y\\z\end{bmatrix})=\begin{bmatrix}x+z\\y-z\\y-x\end{bmatrix}$

(E) None of the above.

9. (5%) Which of the following is an inner product defined on $\mathbb{R}^4$? Note that $\mathbf{u}=(u_1,u_2,u_3,u_4)$ and $\mathbf{v}=(v_1,v_2,v_3,v_4)$ are any given vectors in $\mathbb{R}^4$.

(A) $\langle\mathbf{u},\mathbf{v}\rangle=u_1v_1+u_2v_2+u_3v_3+u_4v_4$.

(B) $\langle\mathbf{u},\mathbf{v}\rangle=4u_1v_1+3u_2v_2+2u_3v_3+u_4v_4$.

(C) $\langle\mathbf{u},\mathbf{v}\rangle=u_1v_1-u_2v_2+u_3v_3+u_4v_4$.

(D) $\langle\mathbf{u},\mathbf{v}\rangle=u_1v_1+u_2v_2+u_4v_4$.

(E) None of the above.

10. (5%) Which of the following sets is an orthogonal basis for the designated vector space?

(A) $\left\{\begin{bmatrix}3\\2\end{bmatrix},\begin{bmatrix}2\\-3\end{bmatrix}\right\}$, for $\mathcal{R}^2$ with $\langle\mathbf{x},\mathbf{y}\rangle=\mathbf{x}^T\mathbf{y}$.

(B) $\{\cos x,\sin x\}$, for $C[0,2\pi]$ with $\langle f,g\rangle=\int_0^{2\pi}f(x)g(x)dx$.

(C) $\{1,x,x^2\}$ for $\mathcal{P}_3$ with $\langle f,g\rangle=\int_{-2}^{2}f(x)g(x)dx$.

(D) $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ for $\mathcal{R}^3$ with $\langle\mathbf{x},\mathbf{y}\rangle=\mathbf{x}^TM\mathbf{y}$ where $M=\begin{bmatrix}1&3&0\\3&1&3\\0&3&1\end{bmatrix}$

(E) None of the above.

11 – 13 題為計算題　請於試卷內之「非選擇題作答區」標明題號依序作答。

11. Find the general solution of the following differential equations: (25 scores)

(a) $\frac{dy(x)}{dx}+y(x)-e^{2x}=0$

(b) $\frac{dy(x)}{dx}-y(x)+e^{2x}y^2(x)=0$

(c) $y^{(4)}(x)-y^{(3)}(x)-2y''(x)+2e^x+8=0$

12. (a) Find the inverse Laplace transforms of (15 scores)

$$\frac{s^2}{s^2-2s+3}$$

(b) Find the Laplace transform of

$$e^t\int_0^t e^{-2\tau}\sinh(t-\tau)\cos\tau\,d\tau$$

13. Solve the partial differential equation (10 scores)

$$\frac{\partial u}{\partial x^2}+2\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}+u$$

試題隨卷繳回