1. (10%) Write a recurrence relation and initial conditions for the number s_n of n-bit strings that do not contain the pattern 010.

2. (10%) Solve the following recurrence relations: $s_n = 10s_{n-1} - 25s_{n-2}$, $s_0 = -7$, $s_1 = -15$.

3. (10%) Use extended Euclidean algorithm to find integers x and y such that 539x+396y=154.

4. (10%) Let a_r be the number of solutions to p+q=r for r≥2, where p and q are primes. Precisely,

$a_r = |\{(p,q): p + q = r \wedge p \text{ and } q \text{ are primes}\}|$. Find a generating function $F(x)$ such that $F(x) = \sum_{r=2}^{\infty} a_r x^r$.

5. You are given a combinatorial circuit.
   (a) (5%) Write a Boolean expression corresponding to the given circuit.
   (b) (5%) Give the truth-table for the given circuit.
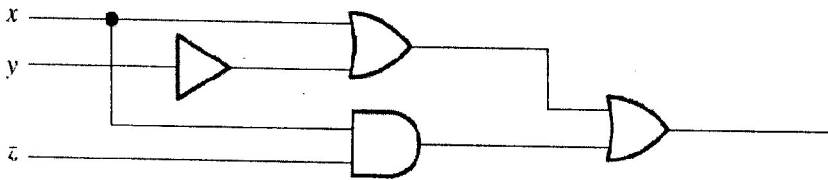
6. (15%) Is the set of real numbers between 0.12468 and 0.12469 a countably infinite set or an uncountably infinite set? Prove your answer!

7. Give a set A = {triangle T, circle C, square S} with the following attributes:

	shape	color	size
triangle T	triangle	green	median
circle C	circle	yellow	big
square S	square	purple	small

   (a) (8%) Design a totally ordered set by using the set A and the above attributes only.
   (b) (12%) Explain the reasons why the objects in the set A can be ordered according to your design.

8. (15%) Discuss Euler′s formula for planar graphs as much as possible.