題號： 396
科目：應用微積分
節次： 2

國立臺灣大學 104 學年度碩士班招生考試試題

題號： 396
共 1 頁之第 1 頁

（答案請寫在答案卷上）

需列計算過程，否則不予計分

填充計算題(總計 10 題，每題 10 分)

1. Let $f(x) = \sqrt{\dfrac{(x+1)^5}{(x^3+2x-1)(x+3)^3}}$. Find $f'(1) = $ ___(1)___ .

2. Find $\lim_{x \to -\infty} \sqrt{x+2}(\sqrt{x+1} - \sqrt{x-3}) = $ ___(2)___ .

3. Let $f(x) = (8-x)(20-4x)x, 0 \le x \le 5$. Find the maximum and minimum values of $f(x)$.

4. Let $f(x) = x^3 + mx - n$ where m and n are real constants and $m > 0$. Show $f(x)$ has at least one real root.

5. Please find $\dfrac{d}{dx} \dfrac{sin^{-1}(x^2)}{(sin^{-1}x)^2} = $ ___(5)___ .

6. Evaluate the following equation: $\lim_{n \to \infty} \sum_{k=1}^{n} \dfrac{n}{k^2 + n^2} = $ ___(6)___ .

7. Evaluate the following equation: $\int_0^\infty \dfrac{\sin ax \sin x}{x^2} dx = $ ___(7)___ if $0 \le a \le 1$.

8. Evaluate the following equation: $\int_0^\infty \dfrac{x \sin ax}{1+x^2} dx = $ ___(8)___ if $a \ne 0$.

9. Evaluate the following equation: $\lim_{n \to \infty} \sum_{k=1}^{n} (n^2 + k^2)^{-\frac{1}{2}} = $ ___(9)___ .

10. Evaluate the following equation: $\dfrac{8}{\pi} \sum_{n=1}^{\infty} \dfrac{n \sin 2nx}{4n^2 - 1} = $ ___(10)___ if $0 < x < \pi$.

試題隨卷繳回