1. Find the general solutions of the differential equations:

(1) $y'+\frac{1}{x+1}y=e^x$ (10%)

(2) $y''-4y'+5y=e^{2x}+e^x\sin(2x)$ (10%)

2. Please use the Laplace transform to solve the initial value problem. (15%)
$y''+3y'+2y=e^t,\quad y(0)=0,\quad y'(0)=0$

3. Find the power series solution of the differential equation, up to 4 non-zero terms. (15%)
$x^2y''-y'+xy=e^x$
$y(-2)=1,\quad y(-2)=6$

4. A *BOD* removal process is carried out in three identical Continuous-flow stirred tank reactors (CFSTRs) in series. The mixing mechanism transfers oxygen at a rate dependent on the oxygen deficit:

$$r_{O_2trans}=k_2\left(C^*-C_{O_2}\right)$$

where, r_{O_2trans} = rate of O_2 transfer, $g/m^3\cdot d$

k_2 = oxygen transfer coefficient, $80\,d^{-1}$

C^* = dissolved – oxygen saturation concentration, $8.00\,g/m^3$

The hydraulic residence time of each tank is 12hr, the influent BOD_u to the first tank is $200\,g/m^3$, and the *BOD* removal rate is first-order:

$$r_{BOD}=-k\left(BOD_u\right),\quad k=10d^{-1}$$

Assuming there is no oxygen in the influent, determine the dissolved – oxygen concentration in the second tank. (20%)

5. $a_0=1,\ a_1=1,\ a_2=2,\ a_3=3,\ a_4=5,\ a_5=8, a_6=13, a_7=21, \ldots.$

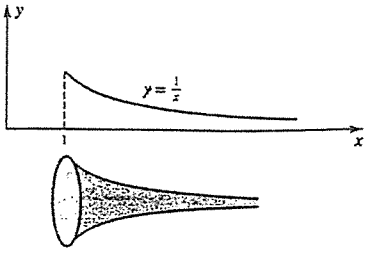
(a) Please find $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\ ?$ (5%)

(b) Please prove $a_n=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1}-\left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$ (5%)

(c) If $F(x)=\frac{1}{1-x-x^2}=a_0+a_1x+a_2x^2+\cdots+a_nx^n+\cdots$

Please find $a_0,\ a_1,\ a_2,\ \ldots,\ a_n,\ \ldots$ (5%)

6. For a Gabriel's horn (or Torricelli's Trumpet), Please find its volume (V) and curved surface area (S). (15%)



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