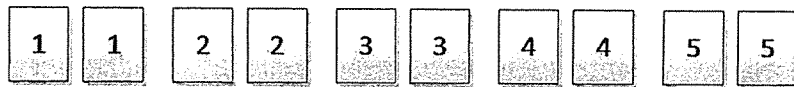


共 9 大題。總分 100 分。

- (13 %) Consider two propositions: $P: (A \vee B) \Rightarrow C$ and $Q: (C \Rightarrow B) \vee (\neg C \Rightarrow \neg A)$. Please draw a truth table (10 %) and decide the relationship between P and Q (3 %). You can use as many columns as you need.
- (12 %) Show that the following statements about the integer x are equivalent: (a) $3x + 2$ is even, (b) $x + 5$ is odd, and (c) x^2 is even.
- (10 %) Among the arrangements of the 26 different English letters: (a) How many ways can we permute to obtain "BRT" or "FAN"? (b) How many ways can we permute such that none of the patterns "USA" and "SAP" occur?

4. (10 %) Suppose that you have ten cards as shown below:



You shuffle them and deal them in a row and you might get:



What is the expected number of adjacent pairs with the same value? In the example, there are two adjacent pairs with the same value, the 3's and the 5's.

- (10 %) Let A be an invertible matrix. Show that $(A^n)^{-1} = (A^{-1})^n$ whenever n is a positive integer.
- (10 %) Prove that if $n \in \mathbb{Z}^+$ and $n \geq 2$, then $\prod_{i=2}^n \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$.
- (10 %) Define the sequence of numbers as follows:
$$a_n = \begin{cases} 1 & \text{if } 0 \leq n \leq 3, \\ a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4} & \text{if } n \geq 4. \end{cases}$$
Prove that $a_n \equiv 1 \pmod{3}$ for all $n \geq 0$.
- (15 %) Define $a_0 = 1$, $a_1 = 3$, $a_2 = 5$, and $a_n = a_{n-1} \cdot a_{n-2}^2 \cdot a_{n-3}^3$.
 - Write an iterative algorithm for finding the n th term of the sequence. (7 %)
 - Same as (a) but write a recursive algorithm instead. (5 %)
 - Which algorithm is more efficient? (3 %)
- (10 %) Let $S = \{2, 16, 128, 1024, 8192, 65536\}$. If four numbers from S are arbitrary selected, show that the multiplication of any two numbers from the four selected numbers equals 131072.

試題隨卷繳回