國立臺北大學 101 學年度碩士班一般入學考試試題

系（所）別： 統計學系

科　　目： 理論統計

☐可 ☑不可使用計算機

1. Let $X_1, X_2, \cdots\cdots, X_n$, $n \in N$, be a random sample from $U(0,\theta)$, $\theta > 0$, distribution, and $U = \max\{X_1, X_2, \cdots\cdots, X_n\}$, $V = \max\{X_1, X_2, \cdots\cdots, X_n\}$.
   (a) (5%) Find the p.d.f. of U.
   (b) (6%) Find the joint p.d.f. of (U,V).
   (c) (5%) For a given $\varepsilon > 0$, evaluate $P(|U-\theta| \geq \varepsilon)$.
   (d) (5%) Show that U is a consistent estimator of θ.
   (e) (9%) If $\theta = 1$, $R = V - U$, $S = \frac{U}{V}$. Find the joint p.d.f. of (R,S).

2. Suppose that the number of customers shopping at the convenient store on campus of National Taipei University each day is a random variable N which has a Poisson distribution with mean λ, $\lambda > 0$. Given $N = n$, the purchase amounts $X_1, X_2, \cdots\cdots, X_n$ of customers are identically independent random variables which have exponential distribution with mean β, $\beta > 0$, and $X_1, X_2, \cdots\cdots, X_n$ are also independent to N. Let the total sale amount of the convenient store per day be $S_N = X_1 + X_2 + \cdots\cdots + X_N$.
   (a) (6%) Given $N = n$, find the conditional pdf of S_n.
   (b) (5%) Calculate $E(S_N)$, the unconditional mean of S_N.
   (c) (9%) Calculate $Var(S_N)$, the unconditional variance of S_N.

3. (20%)請定義隨機變數數列之機率收斂(converge in probability)、分配收斂(converge in distribution)。陳述並證明兩種收斂方式之間的充分或必要關係，並以特例說明兩者之間的非充分或非必要關係。

4. (30%) 假設隨機變樣本 $X_1, X_2, ..., X_n$ 服從 Double exponential 分配，$f(x) = \frac{1}{2} e^{-|x-\mu|}$，其中 μ 為未知參數。令 μ_0 為已知數。
   (a) 請定義最大檢力檢定(the most powerful test)及最大不偏檢力檢定(the most powerful unbiased test)。
   (b) 請問 $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$ 是否有最大檢力檢定或是最大不偏檢力檢定？請證明你的答案，如果有則請找出檢定統計量。
   (c) 請問 $H_0 : \mu \leq \mu_0$ vs. $H_1 : \mu > \mu_0$ 是否有最大檢力檢定或是最大不偏檢力檢定？請證明你的答案，如果有則請找出檢定統計量。

試題隨卷繳交