

國立臺北大學 101 學年度碩士班一般入學考試試題

系(所)別：統計學系

科目：基礎數學

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可 不可使用計算機

注意事項：

1. 考生不得攜帶計算機
2. 請依序作答

一、

1. Let A be an $n \times n$ matrix. Please show that if $\text{Tr}(AA^T) = 0$, then A is a zero matrix. (6%)
2. Let u be an $n \times 1$ matrix such that $u^T u = 1$ and let $H = I_n - 2uu^T$. Please show that $H^{-1} = H^T$. (6%)
3. Let $L: P_3 \rightarrow P_3$ be defined by $L[p(t)] = p''(t) + p(0)$.
 - (a) Let $T = \{t^3, t^2 - 1, t, 1\}$. Show that T is a bases for P_3 . (7%)
 - (b) Show that L is a linear transformation. (7%)
 - (c) Compute the matrix of L with respect to T . (10%)

4. Suppose $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$.

- (a) Please orthogonally diagonalize A . (10%)
- (b) Find the rank of A and nullity of A . (4%)

二、Calculus

1. Find $\iint_A yx^2 dx dy = ?$, where $A = \{(x, y) | x^2 + y^2 \leq 16, y \geq 0\}$. (8%)
2. Let $f(x, y) = \sin^{-1}(3x + 2y)$, and $x = r^3 e^t$, $y = \sin(rt)$. Find $\frac{\partial f}{\partial r} = ?$ (8%)
3. Find $\lim_{x \rightarrow \infty} \frac{e - (1+x)^{\frac{1}{x}}}{x} = ?$ (8%)
4. Find the arc length of the graph $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $[1, 2]$. (8%)
5. Find the integral $\int \frac{x+2}{x^4+x} dx = ?$ (8%)
6. Find $\int_0^\pi \cos^4 \theta d\theta = ?$ (10%)

試題隨卷繳交