

考試科目	微積分 41832	所別	風險管理與保險學系/精 算科學組 4183	考試時間	2 月 28 日(六) 第 1 節
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Please show all your work.

1. (20%) A firm only produces products A and B. Suppose that  $x$  represents the number of product A produced and sold by the firm each year;  $y$  denotes the number of product B. The firm's annual revenue and cost functions are  $F(x, y)$  and  $G(x, y)$  respectively as follows (in New Taiwan Dollars).

$$F(x, y) = -0.2x^2 + 200x - 0.2xy + 160y - 0.25y^2$$

$$G(x, y) = 100x + 70y + 4000$$

- (a) If you are the owner of the firm, how many products A and B you would produce each year to have the maximum profit? (10%)  
 (b) What would the maximum profit be? (10%)

2. (10%) Evaluate  $\int_{-2}^2 \int_{-\sqrt{-x^2+4}}^{\sqrt{-x^2+4}} \int_{\sqrt{x^2+y^2}}^2 (x^2 + y^2) dz dy dx$

3. (10%) Assume  $F(x) = g(f(x))$  and  $g(2) = 23, f(10) = 2, g'(2) = 2$  and  $f'(10) = 44$ . Find  $F'(10)$ .

4. (10%) Find the limit of the following sequence:

$$a_t = \left(1 + \frac{2}{t}\right)^t$$

5. (10%) Assume that the area surrounded by the following two parabolas is 576. Find the values of  $b$ .

$$y = b^2 - x^2$$

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6. (10%) Suppose that  $y + 12 = y^3 x^2 + 6x^2$  and that  $y = 2$  when  $x = 1$ . Find  $\frac{dy}{dx}$ .

7. (10%) Evaluate  $\int_{-3}^3 \frac{x}{1+|x|} dx$ .

8. (20%) The concentration of Drug A in a patient's bloodstream  $t$  minutes after injection is as follows:

$$F(t) = \frac{4t}{1 + 3t^2}$$

- (a) Find the horizontal asymptote of the graph of  $F$ . (10%)  
 (b) What does your finding suggest? (10%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。
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