考試科目	數理統計學 4141	所別	統計	考試時間	2月 28日(六) 第3節

1. (25pts) Let Y~ Uniform(0, 1). Let $X = \theta Y^{1/3}$. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. with distribution same as X.
   (a) (5pts) Find the probability density function of X and the cumulative distribution function of $X_{(n)}$.
   (b) (5pts) Obtain a complete and sufficient statistic for θ.
   (c) (5pts) Obtain the MLE $\hat{\theta}$ of θ.
   (d) (5pts) Find $E(\hat{\theta})$ and derive an unbiased estimator for θ.
   (e) (5pts) Find UMVUE of θ.

2. (10pts) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from Gamma (α, β) distribution, where $\beta > 0$ is the unknown parameter and $\alpha > 0$ is a known constant. Show that $\sqrt{n}(\hat{\beta} - \beta)$ converges to a non-degenerate asymptotic distribution as $n \to \infty$ and identify the distribution.

3. (10pts) Suppose that X has pdf $f(x|\theta) = 2\theta(1-2x) + 2x$ on [0,1] for $\theta \in \Theta = [0,1]$. A Bayesian wants to test $H_0: \theta \leq 0.4$ *vs* $H_a: \theta > 0.4$. If the Bayesian's prior distribution is uniform on [0,1], what is the pearson's (0-1 loss optimal) test?

4. (10pts) Let the random variable X has p.d.f $f(x;\theta) = \frac{1}{\theta^2} x e^{-\frac{x}{\theta}}, x > 0$, (and 0 otherwise), $\theta \in \Omega = (0, \infty)$.
   What are the $E_\theta \tilde{\theta}_n$ *and* $\sigma_\theta(\tilde{\theta}_n)$. $\tilde{\theta}_n$ is the moment estimator of θ, $\tilde{\theta}_n = \tilde{\theta}_n(\mathbf{X}_n)$, $\mathbf{X}_n = (X_1, X_2, \ldots, X_n)$.

5. (45pts) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from the Uniform(0, 1),
   $Y_n = (\prod_{i=1}^{n} X_i)^{-\frac{1}{n}}$, and $Z = X_{(n)} - X_{(1)}$.
   (a)(20pts) Show that $\sqrt{n}(Y_n - e) \Rightarrow N(0, e^2)$.
   (b)(25pts) Derive the probability density function of Z. (20pts). Is Z independent of $X_{(n)}$? (5pts).

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。