國立政治大學 104 學年度碩士班招生考試試題

考試科目	基礎數學 41412	所別	統計學系	考試時間	2 月 28 日(六) 第一節

1. (15 points) Find the following integrals.

   (a) $\int_0^1 3.1^x dx.$

   (b) $\int_0^1 \log_{3.1}(x) dx.$

   (c) $\int_0^1 x^{3.1} dx.$

   (d) $\int_0^1 \sin(x) dx.$

   (e) $\int_0^1 \cos(x) dx.$

2. (20 points) Suppose that f is a differentiable function such that $f(0) = 1$ and

$$f'(x) = \frac{x}{2+\sin(x)}$$

for $x \in (-\infty, \infty)$.

   (a) Find the minimum of f on $(-\infty, \infty)$.

   (b) Show that $\lim_{x\to\infty} f(x) = \infty$.

3. (10 points) Suppose that $a_1 = b_1 = 1$ and for $n \geq 2$,

$$a_n = a_{n-1} + \frac{n}{2+\sin(n)}$$

and $b_n = 1/n$.

   (a) Determine whether $\lim_{n\to\infty} a_n b_n$ exists. Justify your answer.

   (b) Determine whether $\sum_{n=1}^{\infty} a_n b_n$ is finite. Justify your answer.

4. (5 points) Let $D_1 = \{(x,y) : x < 0 \text{ and } y < 0\}$ and $D_2 = \{(x,y) : x > 1 \text{ and } y > 1\}$. Define

$$f(x,y) = \begin{cases} 0 & \text{if } (x,y) \in D_1; \\ x/(1-y+x) & \text{if } y \leq x \text{ and } (x,y) \notin D_1 \cup D_2; \\ y/(1+y-x) & \text{if } y > x \text{ and } (x,y) \notin D_1 \cup D_2; \\ 1 & \text{if } (x,y) \in D_2. \end{cases}$$

Determine whether f is continuous at $(0,0)$. Justify your answer.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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5. (20 points) Suppose that V is a vector space, and three vectors e_1, e_2 and e_3 form a basis for V. Suppose that L: $V \to V$ is a linear transform such that $L(e_1) = e_1 + e_2$, $L(e_2) = e_2$ and $L(e_3) = e_1$. Find the dimension for the space $\{v \in V : L(v) = 0\}$ and the dimension for the range of L. Justify your answers.

6. (20 points) Suppose that A is a 3×3 real matrix with eigenvalues 1, 2, 3 and associated eigenvectors v_1, v_2 and v_3 respectively.

   (a) Can we conclude that v_1, v_2 and v_3 are linearly independent? Justify your answer.

   (b) Suppose that v_1, v_2 and v_3 are orthogonal. Can we conclude that A is symmetric? Justify your answer.

7. (10 points) Suppose that $-1 < a < 1$ and

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 1 & 0 \\ 0 & a & 1 \end{pmatrix}$$

Find the eigenvalues of A. Show your work.

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