# 國立政治大學　104　學年度碩士班招生考試試題



| 考試科目 | 統計學 21613 | 所別 | 經濟學系 | 考試時間 | 3 月 1 日( 日 ) 第 三 節 |
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注意事項:

(1) 請依題號順序作答。

(2) 不可使用計算機。

(3) 答題若過程錯誤 (或沒有過程) 但答案正確, 將以「零分」計算。

1. (25%) The joint density function of $X$ and $Y$ is given by

$$f_{X,Y}(x,y) = \begin{cases} kxy^2, & x-1 \le y \le 1-x, 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

   (1) (5%) Find the value of $k$ that makes this a probability density function.

   (2) (5%) Find $\mathbb{P}(X > Y)$.

   (3) (5%) Derive the **marginal density** of $Y$.

   (4) (5%) Find $\mathbb{P}(Y > 0 | X = 0.75)$.

   (5) (5%) Are the random variables $X$ and $Y$ **independent**? (You should clearly write down the reason.)

2. (15%) A random sample of size $n$ is taken from a population with the probability mass function:

$$p_Y(y) = \theta(1-\theta)^{y-1}, \quad y = 1, 2, 3, \ldots.$$

   (1) (5%) Find the **method of moments estimator** for $\theta$.

   (2) (5%) Find the **maximum likelihood estimator** for $\theta$.

   (3) (5%) Which one is better? (You should clearly write down the details for your criteria and comparisons)

3. (10%) Suppose one observation from the probability density function $f_Y(y) = \theta e^{-\theta y}$, $y > 0$, is to be used to test $H_o : \theta = 1.5$ versus $H_1 : \theta < 1.5$. The decision rule calls for the null hypothesis to be rejected if $y \ge \ln 15$.

   (1) (5%) Find the probability of **Type I error**, $\alpha$.

   (2) (5%) Find the testing **power** as a function of $\theta$.

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
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4. (20 points) Consider the following model for panel data with a time effect $\delta_t$,

$$y_{it} = \alpha_0 + \alpha_1 x_{it} + \alpha_2 \delta_t + e_{it}.$$

Show how $\alpha_1$ can be estimated by the ordinary least squares method using "time-demeaned" procedure.

5. (30 points) Given the model

$$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + u_t,$$

with constraints $\beta_0 + \beta_1 = \alpha$ and $\beta_0 + \beta_2 = -\alpha$, suppose that all the classical linear regression assumptions hold.

(a) As $\alpha$ is unknown, how do you test the constraints in the model?

(b) How would you estimate $\alpha$?

(c) Is your estimator $\hat{\alpha}$ in (b) the best linear unbiased estimator?

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