國立政治大學 104 學年度碩士班招生考試試題

考 試 科 目	計算機數學 81413	所 別	資訊科學系 8141	考 試 時 間	03 月 01 日(日)第三節

PART Ⅰ: 離散數學部分:

1. 單選題(20%)

1.1 Which of the following is the smallest big-O estimate for the function $f(n) = 5\, n \log n + \log(n^3/4) + n \log(4/n^5)$?

(a) $\log n$ (b) n (c) $n \log n$ (d) $\log^3 n$

1.2 Let N={0,1,…} be the set of non-negative integers and $f: N \to N$ a function defined by $f(n) = n^2 - 2n + 1$. Then f(n) is

(a) bijective (b) injective but not surjective (c) surjective but not injective (d) none of the above

1.3 When sorted in increasing order of growth rate, which one of the following functions would be second slowest?

(a) $n^3 + 2n + 5$ (b) $10 \log n^4 + n$ (c) $n \log n$ (d) $n! / 5$

1.4 Which of the following propositional formulas is equivalent to $(p \vee q \vee r) \to s$

(a) $(p\to s) \vee (q \to s) \vee (r \to s)$ (b) $(\neg p \to s) \vee (\neg q \to s) \vee (\neg r \to s)$

(c) $(p\to s) \wedge (q \to s) \wedge (r \to s)$ (d) $(\neg p \to s) \wedge (\neg q \to s) \wedge (\neg r \to s)$

1.5 Given the context-free grammar G=(N,T,S,P) where N={S,A,C} is the set of nonterminals, T={a,b} is the set of terminals, S is the start symbol and the set of productions P is given as follows:

$S \to AC$ $C\to aCb \mid ab$ $A \to a \mid Aa$

Then which of the following strings belongs to the language generated by the grammar G?

(a) ab (b) aabab (c) aabbb (d) aaabb

1.6 Let S be the smallest set of non-negative integers satisfying the following two conditions:

1. $4 \in S$, 2. if $\{\alpha, \beta\} \subseteq S$ then $\alpha \times \beta \in S$.

Then which of the following numbers is not an element of S?

(a) 16 (b) 128 (c) 256 (d) 1024

1.7 Let A, B and C be arbitrary sets and - and ~ denote the difference and complement of sets. Then which of the following is a superset of $(A - B) \cup ((\sim A) \cap B \cap C)$

(a) A (b) $A \cup C$ (c) B (d) $(\sim B) \cap A \cap C$

1.8 Which of the following graphs contains a Euler circuit?

(a) K_6 (A complete simple graph with 6 vertices)

(b) W_6 (A wheel with 6 border vertices and one centered vertex)

(c) Q_6 (A 6-dimensional hypercube)

(d) $K_{6,6}$ (A complete bipartite graph with 6 vertices in each block)

1.9 What is the value of the summation $\Sigma_{1\le k\le n}\, k \times C(n,k)$ when n = 10 ?

(a) $2^9 * 10$ (b) $2^{10} * 10$ (c) $2^9 * 100$ (d) $2^{10} * 100$

1.10 Let R be the relation {(2,1),(2,3),(3,2),(3,4),(4,1),(4,3)} on the set{1,2,3,4}. Then which of the following pairs is not in the transitive closure of R relation?

(a) (1,1) (b) (2,2) (c) (3,3) (d) (4,4)

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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2. 填充題 (20%)

2.1 The evaluated value of the postfix expression: 8 2 3 * - 4 * 9 3 / + is ___________

2.2 The coefficient of x^5 in the expansion of the function $(1+x+x^2+x^3+\ldots)^4$ is ______ .

2.3 (5%) Let h : N → N be a function on the set N of non-negative integers defined inductively by h(0) = 3 and h(n+1) = 2 × h(n) + n×n for n ≥ 0. Suppose we are given a higher-order function prim_rec : (N × ((N×N)→N) × N) → N defined as follows:

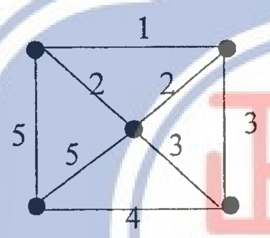
prim_rec (k, f, n) = if (n = 0) then k else f (n-1, prim_rec(k, f, (n-1))).

Find a number K and a binary function F : (N× N) → N such that for all n,

h(n) = prim_rec(K, F, n).

Ans: K= _________ (2%) and F(x,y) = _______________ (3%).

2.4 The weight of any minimal spanning tree of the following graph is ___________



2.5 The chromatic number of the n-dimensional hypercube Q_n for n > 1 is _____________.

2.6 Let G be a regular graph in which every vertex has the same degree. If G has 10 vertices and 30 edges, then every vertex has degree __________.

3. (10%) A binary relation R is called circular if for all elements a, b and c, aRb and bRc imply that cRa. Show that

(a) if R is an equivalence relation then R is circular, and

(b) if R is reflexive and circular, then R is an equivalence relation.

4. (10%)

(a) Show that if n > 1 is an integer, then n does not divide 2^n -1. Hint: Prove it by contradiction. Suppose n divides 2^n -1 and let p be the least prime factor of n. Then we would have the facts that $2^n \equiv 1$ (mod p) and $2^{p-1} \equiv 1$ (mod p) [Fermat's little theorem], from which a contradiction can be derived.

(b) Use the above fact and the pigeon-hole principle to show that if n > 1 is an odd number, then n must divide some number in the set $\{2^1-1, 2^2-1, \ldots, 2^{n-1}-1\}$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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Part II. Linear Algebra (40%)

5. [10%] Please show that if A is any real square matrix then $\mathbf{S} = \frac{1}{2}(\mathbf{A}+\mathbf{A}^{T})$ is symmetric and $\mathbf{T} = \frac{1}{2}(\mathbf{A}-\mathbf{A}^{T})$ is skew-symmetric.

6. [10%] (1) Please find a real 2x2 matrix $A \neq \mathbf{I}$, the identity matrix or unit matrix, such that $A^2 = \mathbf{I}$. (2) Please find a real 2x2 matrix $A \neq 0$ such that $A^2 = 0$.

7. [10%] Please show that if $\mathbf{x}$ is an eigenvector of a matrix A corresponding to an eigenvalue λ, so is $k\mathbf{x}$ with any $k \neq 0$.

8. [10%] Please show that the determinant of an orthogonal matrix has the value +1 or -1.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。