國立嘉義大學 104 學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學(條件不足，請自行做合理假設。)

1. Solve the following ordinary differential equation. (20%)

$$y''' - 6y'' + 11y' - 6y = e^x$$

2. Show that the vector field $\vec{F} = (yz+2y)\vec{i} + (xz+2x)\vec{j} + (xy+3)\vec{k}$ is conservative. If P_1 is the origin $(0,0,0)$ and P_2 is the point $(1,1,1)$, find line integral $\int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$

   (a) by using the associated scalar potential Φ, (10%).

   (b) by evaluation of the line integral along the straight-line path between P_1 and P_2, (5%).

   (c) find another path to show path independence of the line integral. (5%)

3. Find the eigenvalue and eigenvector of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ (20%)

4. Solve the initial-value problem by the Laplace transformation.(20%)

$$y'' + 2y' + y = e^{-t} \ , \qquad y(0) = -1 \ , \quad y'(0) = 1$$

5. Find all unit vectors $\vec{a} = [a_1, \ a_2]$ in the plane orthogonal to $[2, \ 3]$. (20%)