系所：

科目：工程數學
考試時間：100 分鐘

土木與環境工程學系（土木工程組）

本科原始成績：100 分

是否使用計算機：是

---

1. (20%)

$A = \begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$, there exists a diagonal matrix $D = P^T A P$, Determine P=? and D=?

2. (30%)

A fluid with the velocity field of $v = y^3 \vec{i} - x^3 \vec{j} + 0\vec{k}$. Show (a) the flowing path and determine (b) whether the fluid is compressible, (c) the flowing field is rotational?

3. (15%)

Use the method of change valuables (i.e. $u = y'$) to find the solution to the differential equation, $y' + xy'' = \ln |x|$.

4. (15%)

Use the method of power series (i.e. $y = \sum_{n=0}^{\infty} a_n x^n$) to find the solution to the differential equation, $y^{(4)} + x^2 y' - 2y = 0$.

5. (20%)

Use Laplace Transform to find the solution to the system differential equations, $\begin{cases} x' + y' + x - y = 0 \\ x' + 2y' + x = 1 \end{cases}$ and $\begin{cases} x(0) = 0 \\ y(0) = 0 \end{cases}$.