國立臺灣科技大學101學年度碩士班招生試題

系所組別：電機工程系碩士班丁二組

科　　目：控制系統

(總分為100分)

***Problem 1:* (*20 points*)**【證明題】

The unilateral Laplace transform is defined as shown below

$$L\{f(t)\} = F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt,$$

and define $f(0^-)$ be the pre-initial value and $f(0^+)$ the post-initial value of $f(t)$ at the initial time $t=0$.

(a) Prove that $L\{f'(t)\} = sF(s) - f(0^-)$. **(*7 points*)**

(b) Prove that $f(0^+) = \lim_{s\to+\infty} sF(s)$. **(*7 points*)**

(c) Prove that $L\{\int_{0^-}^{t} f(\tau)d\tau\} = \frac{F(s)}{s}$. **(*6 points*)**

***Problem 2:* (*30 points and 3 points each*)**

Answer each question "Yes" or "No". Just write down your answer, there is no need to specify the seasons.【是非題：只需回答是(Yes)或否(No)，無需說明任何緣由或計算過程。請將是非題答案填寫於答案紙內，於試題紙上作答不予計分】

【 】(1) A system with input $u(t)$ and output $y(t) = |u(t)|$ is a linear system.

【 】(2) A system with input $u(t)$ and output $y(t) = u(t-T), T>0$ is a causal system.

【 】(3) Given the Laplace transform $L\{f(t)\} = F(s) = \frac{a}{s(s^2+a)}, a>0$, then $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = 1$.

【 】(4) For a linear time-invariant (LTI) system, the system's step response is the inverse Laplace transform of the system transfer function.

【 】(5) For a linear time-invariant system, if it is BIBO (bounded-input bounded-output) stable, then it is also asymptotically stable.

【 】(6) Given a linear time-invariant stable type-1 unity negative-feedback system, the steady state error of a step response is zero.

For Problems (7~10), let's consider an unity negative-feedback system with loop transfer function $L(s) = \frac{K}{(s+2)(s+10)}, K>0$.

【 】(7) If the gain K is increasing, then the gain margin GM is decreasing.

【 】(8) If the gain K is increasing, then the phase margin PM is also increasing.

【 】(9) If the gain K is increasing, then the gain crossover frequency ω_{cg} is also increasing.

【 】(10) The closed-loop system is an over-damped system for all $K>0$.

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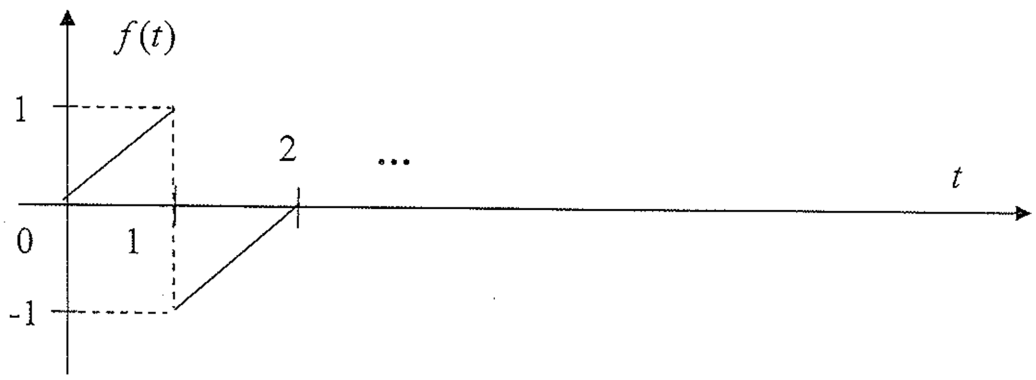
Problem 3: **(13 points)**

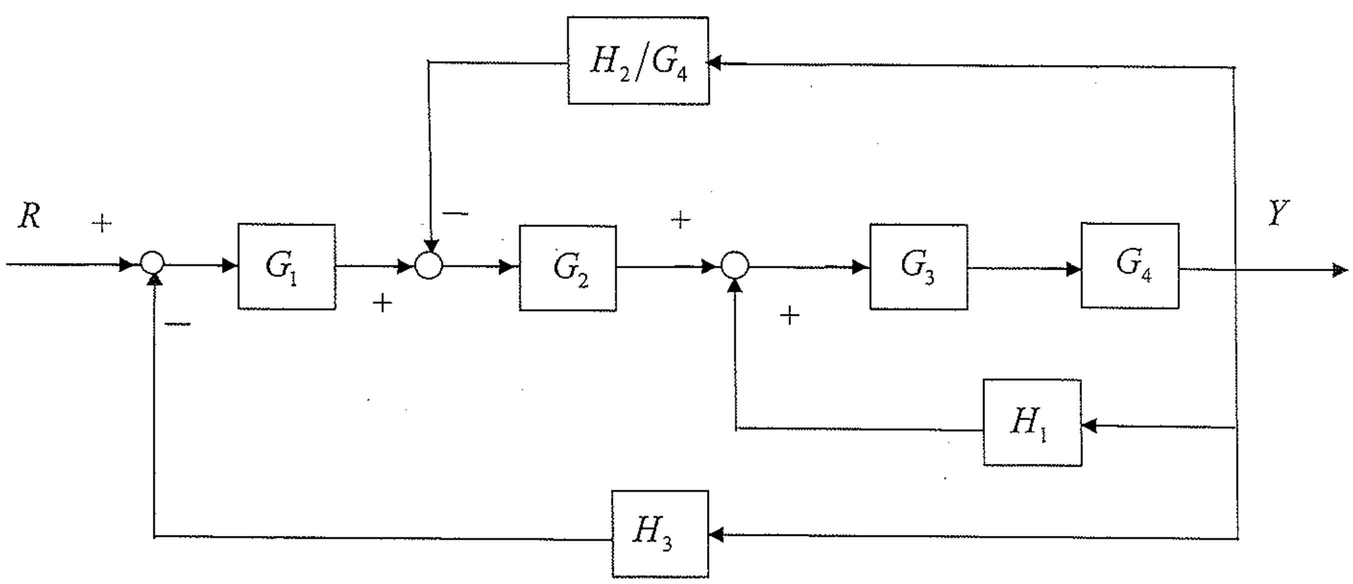
Consider the following periodic function $f(t)$ in one period described as follows:



Find its Laplace transform (i.e., $F(s)$).

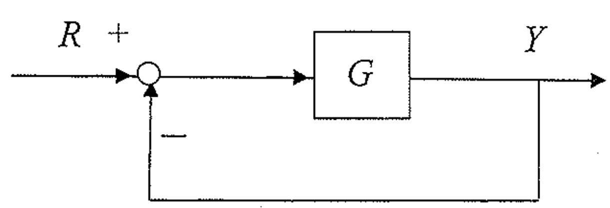
Problem 4: **(13 points)**

Find the relation between $R(s)$ and $Y(s)$.



Problem 5: **(24 points)**

Consider a negative unit feedback system with forward transfer function $G(s)=ke^{-0.2s}/\left[s(1+0.1s)\right]$. (a) For $k=2.5$, find the GM (gain margin), PM (phase margin), ω_g (gain-crossover frequency) and ω_p (phase-crossover frequency) (16 points). (b) Determine the value of k such that the closed-loop system is unstable (8 points).



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