國立臺灣科技大學101學年度碩士班招生試題

系所組別：電子工程系碩士班乙一組

科　　目：通信系統

（總分為100分）

1. The joint probability function of two discrete random variables $X$ and $Y$ is given below:

$$\Pr\text{ob}(X=x,Y=y)=\begin{cases}\frac{1}{2} & \text{if}(x,y)=(0,0),\\ \frac{1}{4} & \text{if}(x,y)=(0,1),\\ \frac{1}{20} & \text{if}(x,y)=(1,0),\\ \frac{1}{5} & \text{if}(x,y)=(1,1),\\ 0 & \text{otherwise}.\end{cases}$$

(a) (3%) Let $\text{Var}(X)$ denote the variance of $X$. Then, $\text{Var}(X)=?$

(b) (3%) Let $\text{E}(Y)$ denote the mean of $Y$. Then, $\text{E}(Y)=?$

(c) (3%) Given the condition of $X=1$, what is the probability of $Y=0$?

In other words, $\Pr\text{ob}(Y=0 \mid X=1)=?$

(d) (3%) Given the condition of $Y=0$, what is the probability of $X=1$?

In other words, $\Pr\text{ob}(X=1 \mid Y=0)=?$

(e) (3%) Are $X$ and $Y$ independent?

2. The Fourier transform of $x(t)$ is defined as

$$X(f)=\int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}\,dt$$

It is known that the Fourier transform of $e^{-\pi t^2}$ is $e^{-\pi f^2}$.

(a) (5%) Find the Fourier transform of $\frac{1}{\sqrt{2\pi}}e^{-t^2/2}$.

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(b) (5%) Find the Fourier transform of $\cos(2\pi f_c t)\cdot e^{-t^2/2}$, where $f_c$ is a given constant.

3. (10%) In this problem, let us consider data transmission over an AWGN channel. Let $E_b$ denote the energy (measured in joule) consumed in the transmission of one data bit. Let $N_0/2$ denote the two-sided power spectral density (measured in watts/Hz) of the Gaussian noise. Please sort the four cases listed below according to their bit-error-rate performance (from the best to the worst).

(A) BPSK, with $E_b/N_0 = 10\text{dB}$

(B) coherent BFSK, with $E_b/N_0 = 12\text{dB}$

(C) noncoherent BFSK, with $E_b/N_0 = 12\text{dB}$

(D) QPSK, with $E_b/N_0 = 11\text{dB}$

4. Let us consider BPSK demodulation in this problem. The transmitter either sends out $+\sqrt{E_b}$ or sends out $-\sqrt{E_b}$. In the channel, the transmitted signal is corrupted by an additive noise W, which is distributed as zero-mean Gaussian with a variance of $N_0/2$. In other words, the received signal is $z = \pm\sqrt{E_b} + W$ (obviously, the sign of $\sqrt{E_b}$ depends on what had been sent by the transmitter). The decision rule is:

$$\text{Decide that } +\sqrt{E_b} \text{ was sent if } z > \tau ,$$

where $\tau$ is some threshold value. Let us adopt the MAP (i.e.

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maximum a-posteriori) criterion for the determination of $\tau$. Assume that we have the a-*priori* information: $\Pr ob(\text{Transmitter sends out } +\sqrt{E_b})$ $= 2/3$

(a) (4%) Which of the three cases below is correct: (A) $\tau > 0$, (B) $\tau = 0$, (C) $\tau < 0$ ?

(b) (8%) Find the exact value of $\tau$. Please express your answer in terms of $E_b$ and $N_0$.

(c) (3%) Let $\Pr ob(\text{error} \mid \text{A was sent})$, where A is either $+\sqrt{E_b}$ or $-\sqrt{E_b}$, denote the probability of error in decision when A was sent by the transmitter. Is it true that $\Pr ob(\text{error} \mid +\sqrt{E_b}$ was sent$) > \Pr ob(\text{error} \mid -\sqrt{E_b}$ was sent$)$?

5. (10%) Please prove that the single-sideband modulated signal can be expressed as $\frac{1}{2}m(t)\cos(2\pi f_c t) \pm \frac{1}{2}\hat{m}(t)\sin(2\pi f_c t)$, where $m(t)$ is the message signal, $\hat{m}(t)$ is the Hilbert transform of $m(t)$, and $f_c$ is the center frequency.

6. (10%) Suppose that the message signal $m(t) = 10^3 \operatorname{sinc}^2(10^4 t)$ is FM modulated with the frequency sensitivity equal to 10, please find the bandwidth of the modulated FM signal.

7. Suppose that the bandwidth is 1MHz, the center frequency is 900MHz, and QPSK, 16-QAM, and 64-QAM, whose bit error rates (BERs) are $10^{-(4+\mathrm{SNR})}$, $10^{-(3+\mathrm{SNR})}$, and $10^{-\mathrm{SNR}}$, respectively, are used for transmission, answer following questions:

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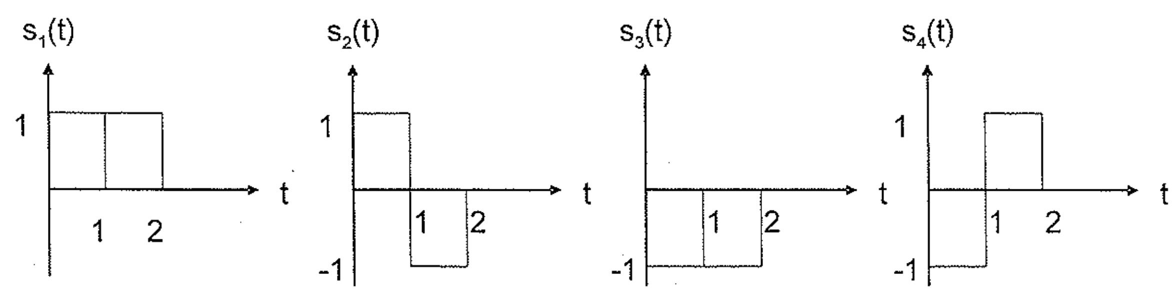
(a) (10%) Suppose that SNR=2dB and BER requirement is $BER<10^{-4}$, which modulation can be used for transmission?

(b) (10%) Based the result of (a), find the maximum transmission bit rate if there is no inter-symbol-interference.

8. (10%) Suppose that the received signal is $r(t)=s_k(t)+n(t)$, where $0\le t\le 2$. The $s_k(t)$ for $1\le k\le 4$ is shown in the figure below and n(t) is white Gaussian with zero mean and two-sided power spectral density $N_0/2$, please compute the symbol error rate.




Figure