

國立臺灣科技大學101學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科目：工程數學

(總分為100分)

1. Find the general solution of $X' = \begin{pmatrix} -2 & 6 \\ 0 & 1 \end{pmatrix} X$ (20%)

2. Find the general solution of $y'' - 3y' + 2y = xe^{2x}$ (20%)

3. Find the solution of the system by using Laplace Transform (20%)

$$\begin{cases} y_1''(t) + y_1(t) + y_2(t) = 1 \\ y_2'(t) - y_1(t) - y_2(t) = 0 \end{cases} \quad y_1(0) = y_1'(0) = y_2(0) = 0$$

4. Verify Gauss theorem: given a vector field

$$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$$

and region M bounded by the hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$ (20%)

Hint: calculate each side of Gauss theorem, respectively, and show that both results are identical.

5. Use Fourier Transform to solve the equation:

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \quad (-\infty < x < \infty, t \geq 0) \text{ with the given conditions}$$

$$y(x, 0) = 4[H(t-2) - H(t-10)], \text{ and } \frac{\partial y}{\partial t}(x, 0) = 0 \quad (20\%)$$

