國立臺北科技大學 101 學年度碩士班招生考試

系所組別：3620 生化與生醫工程研究所乙組

第二節　工程數學　試題

注意事項：

1. 本試題共 5 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Find the general solution for y.

(a) $\dfrac{dy}{dx}=\dfrac{x^2-y}{x(\ln x)}$　(10%)

(b) $x^2y''-4xy'-(x^2-6)y=0$　(15%)

2. Solve the following the differential equation for $y(t)$ using Laplace transform.

$y''+3y'+2y=e^{-t}+e^{-(t-3)}u(t-3)$, $y(0)=0$, $y'(0)=0$

where $u(t)$ is the unit step function.　(20%)

3. (a) Find the Fourier series of the function $f(x)=\dfrac{2}{3}\pi-|x|$ for $-\pi\le x\le\pi$.　(15%)

(b) Use the answer of (a) to find the sum of infinite series $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\dfrac{1}{7^2}+\cdots$　(5%)

4. Consider the line integral $\int_C \vec{F}\bullet d\vec{r}$, where $\vec{r}$ is the position vector,

$\vec{F}=-2y^3\,\vec{i}+\left(2x^3+\cos y\right)\vec{j}$, C: $x^2+y^2=1$, $z=0$　(clockwise).

(a) Is the line integral independent of path? Why?　(5%)

(b) Evaluate the line integral.　(10%)

5. Solve the following partial differential equation for T:

$\dfrac{\partial^2T}{\partial x^2}+\dfrac{\partial^2T}{\partial y^2}=0$　for $0\le x\le 2$, $y\ge 0$

with boundary conditions $T(0,y)=T(2,y)=0$, $T(x,0)=1$ and $T(x,\infty)=0$.　(20%)