

國立臺北科技大學 101 學年度碩士班招生考試

系所組別：3510 化學工程研究所甲組

第三節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve $\frac{d^2y}{dt^2} - 6y^2 = 0$, $y(0) = 1$ and $\frac{dy}{dt}(0) = 2$. (15%)

2. Find the general solution for y .

$$y'' + 4(e^{4x} - 1)y' + 4e^{8x}y = 0 \quad (\text{Hint: Let } t = 2x) \quad (15\%)$$

3. Use the Laplace transform to solve the differential equation for $y(t)$.

$$ty'' + 2y' - (t+2)y = 2e^{-t}, \quad y(0) = 0. \quad (15\%)$$

4. (a) Find the Fourier series of the function $f(x) = \frac{\pi}{2} - |x|$ for $-\pi \leq x \leq \pi$. (10%)

(b) Use the answer of (a) to find the sum of infinite series $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \dots$ (5%)

5. Assume S is the surface of $z = x^2 + y^2$, $z < 1$. Verify the Stokes's theorem if

$$\vec{F} = 3y\vec{i} - xz\vec{j} + yz^2\vec{k}. \quad (\text{Hint: Evaluate the line integral and surface integral}) \quad (20\%)$$

6. Find the eigenvalues and corresponding eigenfunctions of the Sturm-Liouville problem:

$$y'' + (\lambda - 1)y = 0, \quad y(0) + y'(0) = 0, \quad y(\pi) + y'(\pi) = 0 \quad (20\%)$$