國立臺北科技大學 101 學年度碩士班招生考試

系所組別：2403 光電工程系碩士班

第三節　近代物理　試題（選考）

注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

$$\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4}$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4}$$

Planck constant $h = 6.63 \times 10^{-34}$ J·s $= 4.136 \times 10^{-15}$ eV·s

speed of light in free space $c = 3 \times 10^8$ m/s

electron charge $e = -1.6 \times 10^{-19}$ C

proton rest mass $m_p = 1.67 \times 10^{-27}$ kg $= 0.938$ GeV/c^2

the ground-state energy of the hydrogen atom $E_1 = -13.6$ eV

1. A particle with the rest energy of 100MeV is moving at the speed of 0.90c.
   (a) Find its rest mass (in kg) and the kinetic energy (in MeV). (10%)
   (b) Calculate the corresponding de Broglie wavelength (in m). (5%)
   (c) What is the minimum lifetime at rest the particle must have to finish a 500m-long trip? (5%)

2. 1.0 mW of 500 nm light is directed at a photoelectric cell. If 0.2% of the incident photons produce photoelectrons, find the current in the cell. (10%)

3. A measurement establishes the position of a proton with an accuracy of $\pm 1.00 \times 10^{-10}$ m. Find the uncertainty in its position 1.00 s later. Assume $v << c$. (10%)

4. A beam of 12.5-eV electrons is used to bombard gaseous hydrogen.
   (a) How many spectral lines will be emitted? (10%)
   (b) What is the shortest wavelength among the lines? (10%)

5. A particle of mass m moves one directionally in the potential
   $U = \infty, x < 0$
   $U = 0, 0 \le x \le L$
   $U = U_0, x > L$
   The total energy $E < U_0$.
   (a) Show that the bound state energies are given by the equation

$$\tan\left(\frac{\sqrt{2mE}L}{\hbar}\right) = -\sqrt{\frac{E}{U_0 - E}} \quad (20\%)$$

   (b) For fixed L, there is a minimum value of U_0 below which there are no bound states. Find this minimum value of U_0. (10%)

6. What are the possible z components of the orbital angular momentum for an electron in a 4p state? (10%)